Ice Characteristics and Ice/Structure Interactions

September 2010

Guidance Note
NI 565 DT R00 E

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MARINE DIVISION
GENERAL CONDITIONS

ARTICLE 1

1.1. - BUREAU VERITAS is a Society the purpose of whose Marine Division (the "Society") is the classification (" Classification ") of any ship or vessel or structure of any type or part of it or system therein collectively hereinafter referred to as a "Unit" whether linked to shore, river bed or sea bed or not, whether operated or located at sea or in inland waters or partly on land, including submarines, hovercrafts, drilling rigs, offshore installations of any type and of any purpose, their related and ancillary equipment, subsea or not, such as well head and pipelines, mooring legs and mooring points or otherwise as decided by the Society.

The Society:

- prepares and publishes Rules for classification, Guidance Notes and other documents ("Rules");
- issues Certificates, Attestations and Reports following its interventions ("Certificates");
- publishes Registers.

1.2. - The Society also participates in the application of National and International Regulations or Standards, in particular by delegation from different Governments. Those activities are hereafter collectively referred to as " Certification ".

1.3. - The Society can also provide services related to Classification and Certification such as ship and company safety management certification; ship and port security certification, training activities; all activities and duties incidental thereto such as documentation on any supporting means, software, instrumentation, measurements, tests and trials on board.

1.4. - The interventions mentioned in 1.1., 1.2. and 1.3. are referred to as " Services ". The party and/or its representative requesting the services is hereinafter referred to as the " Client ". **The Services are prepared and carried out on the assumption that the Clients are aware of the International Maritime and/or Offshore Industry (the "Industry") practices.**

1.5. - The Society is neither and may not be considered as an Underwriter, Broker in ship's sale or chartering, Expert in Unit's valuation, Consulting Engineer, Controller, Naval Architect, Manufacturer, Shipbuilder, Repair yard, Charterer or Shipowner who are not relieved of any of their expressed or implied obligations by the interventions of the Society.

ARTICLE 2

2.1. - Classification is the appraisement given by the Society for its Client, at a certain date, following surveys by its Surveyors along the lines specified in Articles 3 and 4 hereafter on the level of compliance of a Unit to its Rules or part of them. This appraisement is represented by a class entered on the Certificates and periodically transcribed in the Society's Register.

2.2. - Certification is carried out by the Society along the same lines as set out in Articles 3 and 4 hereafter and with reference to the applicable National and International Regulations or Standards.

2.3. - **It is incumbent upon the Client to maintain the condition of the Unit after surveys, to present the Unit for surveys and to inform the Society without delay of circumstances which may affect the given appraisement or cause to modify its scope.**

2.4. - The Client is to give to the Society all access and information necessary for the safe and efficient performance of the requested Services. The Client is the sole responsible for the conditions of presentation of the Unit for tests, trials and surveys and the conditions under which tests and trials are carried out.

ARTICLE 3

3.1. - **The Rules, procedures and instructions of the Society take into account at the date of their preparation the state of currently available and proven technical knowledge of the Industry. They are not a standard or a code of construction neither a guide for maintenance, a safety handbook or a guide of professional practices, all of which are assumed to be known in detail and carefully followed at all times by the Client.**

Committees consisting of personalities from the Industry contribute to the development of those documents.

3.2. - The Society only is qualified to apply its Rules and to interpret them. Any reference to them has no effect unless it involves the Society's intervention.

3.3. - The Services of the Society are carried out by professional Surveyors according to the applicable Rules and to the Code of Ethics of the Society. Surveyors have authority to decide locally on matters related to classification and certification of the Units, unless the Rules provide otherwise.

3.4. - **The operations of the Society in providing its Services are exclusively conducted by way of random inspections and do not in any circumstances involve monitoring or exhaustive verification.**

ARTICLE 4

4.1. - The Society, acting by reference to its Rules:

- reviews the construction arrangements of the Units as shown on the documents presented by the Client;
- conducts surveys at the place of their construction;
- classes Units and enters their class in its Register;
- surveys periodically the Units in service to note that the requirements for the maintenance of class are met.

The Client is to inform the Society without delay of circumstances which may cause the date or the extent of the surveys to be changed.

ARTICLE 5

5.1. - The Society acts as a provider of services. This cannot be construed as an obligation bearing on the Society to obtain a result or as a warranty.

5.2. - The certificates issued by the Society pursuant to 5.1. here above are a statement on the level of compliance of the Unit to its Rules or to the documents of reference for the Services provided for.

In particular, the Society does not engage in any work relating to the design, building, production or repair checks, neither in the operation of the Units or in their trade, neither in any advisory services, and cannot be held liable on those accounts. Its certificates cannot be construed as an implied or express warranty of safety, fitness for the purpose, seaworthiness of the Unit or of its value for sale, insurance or chartering.

5.3. - The Society does not declare the acceptance or commissioning of a Unit, nor of its construction in conformity with its design, that being the exclusive responsibility of its owner or builder, respectively.

5.4. - The Services of the Society cannot create any obligation bearing on the Society or constitute any warranty of proper operation, beyond any representation set forth in the Rules, of any Unit, equipment or machinery, computer software of any sort or other comparable concepts that has been subject to any survey by the Society.

ARTICLE 6

6.1. - The Society accepts no responsibility for the use of information related to its Services which was not provided for the purpose by the Society or with its assistance.

6.2. - If the Services of the Society cause to the Client a damage which is proved to be the direct and reasonably foreseeable consequence of an error or omission of the Society, its liability towards the Client is limited to ten times the amount of fee paid for the Service having caused the damage, provided however that this limit shall be subject to a minimum of eight thousand (8,000) Euro, and to a maximum which is the greater of eight hundred thousand (800,000) Euro and one and a half times the above mentioned fee.

The Society bears no liability for indirect or consequential loss such as e.g. loss of revenue, loss of profit, loss of production, loss relative to other contracts and indemnities for termination of other agreements.

6.3. - All claims are to be presented to the Society in writing within three months of the date when the Services were supplied or (if later) the date when the events which are relied on of were first known to the Client, and any claim which is not so presented shall be deemed waived and absolutely barred. Time is to be interrupted thereafter with the same periodicity.

ARTICLE 7

7.1. - Requests for Services are to be in writing.

7.2. - **Either the Client or the Society can terminate as of right the requested Services after giving the other party thirty days' written notice, for convenience, and without prejudice to the provisions in Article 8 hereunder.**

7.3. - The class granted to the concerned Units and the previously issued certificates remain valid until the date of effect of the notice issued according to 7.2. here above subject to compliance with 2.3. here above and Article 8 hereunder.

7.4. - The contract for classification and/or certification of a Unit cannot be transferred neither assigned.

ARTICLE 8

8.1. - The Services of the Society, whether completed or not, involve, for the part carried out, the payment of fee upon receipt of the invoice and the reimbursement of the expenses incurred.

8.2. Overdue amounts are increased as of right by interest in accordance with the applicable legislation.

8.3. - The class of a Unit may be suspended in the event of non-payment of fee after a first unfruitful notification to pay.

ARTICLE 9

9.1. - The documents and data provided to or prepared by the Society for its Services, and the information available to the Society, are treated as confidential. However:

- clients have access to the data they have provided to the Society and, during the period of classification of the Unit for them, to the **classification file** consisting of survey reports and certificates which have been prepared at any time by the Society for the classification of the Unit;
- copy of the documents made available for the classification of the Unit and of available survey reports can be handed over to another Classification Society, where appropriate, in case of the Unit's transfer of class;
- the data relative to the evolution of the Register, to the class suspension and to the survey status of the Units, as well as general technical information related to hull and equipment damages, are passed on to IACS (International Association of Classification Societies) according to the association working rules;
- the certificates, documents and information relative to the Units classed with the Society may be reviewed during certificating bodies audits and are disclosed upon order of the concerned governmental or inter-governmental authorities or of a Court having jurisdiction.

The documents and data are subject to a file management plan.

ARTICLE 10

10.1. - Any delay or shortcoming in the performance of its Services by the Society arising from an event not reasonably foreseeable by or beyond the control of the Society shall be deemed not to be a breach of contract.

ARTICLE 11

11.1. - In case of diverging opinions during surveys between the Client and the Society's surveyor, the Society may designate another of its surveyors at the request of the Client.

11.2. - Disagreements of a technical nature between the Client and the Society can be submitted by the Society to the advice of its Marine Advisory Committee.

ARTICLE 12

12.1. - Disputes over the Services carried out by delegation of Governments are assessed within the framework of the applicable agreements with the States, international Conventions and national rules.

12.2. - Disputes arising out of the payment of the Society's invoices by the Client are submitted to the Court of Nanterre, France.

12.3. - **Other disputes over the present General Conditions or over the Services of the Society are exclusively submitted to arbitration, by three arbitrators, in London according to the Arbitration Act 1996 or any statutory modification or re-enactment thereof. The contract between the Society and the Client shall be governed by English law.**

ARTICLE 13

13.1. - **These General Conditions constitute the sole contractual obligations binding together the Society and the Client, to the exclusion of all other representation, statements, terms, conditions whether express or implied. They may be varied in writing by mutual agreement.**

13.2. - The invalidity of one or more stipulations of the present General Conditions does not affect the validity of the remaining provisions.

13.3. - The definitions herein take precedence over any definitions serving the same purpose which may appear in other documents issued by the Society.

BV Mod. Ad. ME 545 k - 17 December 2008

Contents

1. INTRODUCTION

1.1. Purpose of the document

The purpose of this document is to collect and provide data on the ice (physical and mechanical characteristics) as well as giving some guidance on the calculations of the forces generated by the ice on ships and offshore structures.

The Chapters 2 and 3 of the present document give respectively some information on the different types of ice and on the mechanical properties of these different types of ice.

The Chapter 4 gives some analytical formulae and methodologies to estimate the forces applied on the structures due to ice, with respect to the different modes of failure of the ice.

These pressures and loads may be used to assess the strength of the structure.

1.2. Need for direct calculations

The design of a ship or offshore unit for operation in ice infested waters requires a particular attention to ensure it will be able to fit for the harsh environment due to ice, snow, cold temperature, freezing...

To protect ships and offshore units operated in ice environment, suitable reinforcements and considerations are necessary.

For this purpose, Bureau Veritas Rules and Regulations for Steel Ships and for Offshore Units propose different classification notations:

- ICE CLASS notations [1] for navigation in first year ice, which mean that the ice is accumulated during one winter season. These conditions are typical of the Baltic Sea or the Saint Lawrence Gulf. ICE CLASS notations range decreasingly from **ICE CLASS IA SUPER** to **ICE CLASS IC**, and are based on *Finish - Swedish ice class rules* [2]. These notations provide requirements for both hull strengthening and propulsion.
- POLAR CLASS notations [3] for navigation in Arctic and Antarctic waters which can vary from thick first-year ice to multi-year ice. POLAR CLASS notations range decreasingly from **POLAR CLASS 1** to **POLAR CLASS 7** and are based on *IACS Unified Requirements concerning Polar Class* [4]. These notations provide requirements for both hull strengthening and propulsion.
- ICEBREAKER notations [3] for independent navigation in Arctic and Antarctic waters which can vary from thick first-year ice to multi-year ice. ICEBREAKER notations range decreasingly from **ICEBREAKER 1** to **ICEBREAKER 7** and are based on POLAR CLASS notations. These notations provide requirements for both hull strengthening and propulsion.
- **COLD** notation [1] for cold temperature, de-icing, snow and ice accretion. **COLD** notation includes requirements or steel grade selection for the hull and equipment, freezing prevention of ballast, additional power installation for heating, de-icing…

Generally, the ice notation is selected by the Owner or the Operator. The selection of a suitable ice class notation is the result of an analysis which takes into account various parameters. Some of these parameters deal with the mode of operation of the ship:

- area of navigation,
- time of the year at which the navigation is intended,
- Operational conditions (independent, channel or icebreaker convoy navigation).

Some other parameters concern the economical aspects of the ship exploitation, where a balance is to be found between these different parameters:

- reduction of the icebreaking fees,
- additional steel weight for the reinforcements,
- additional cost for sufficient power propulsion,
- Additional oil consumption.

And last but not least, the local authorities' requirements are to have a major influence on the choice of the most suitable ice class notation, since the notations provide a standard of reinforcement easily identified by charterers, insurers, and port and flag authorities.

To help Owners, Shipyards and Operators in the selection of a suitable ice notation, Bureau Veritas issued Guidance Note NI543 "Ice Reinforcement Selection in Different World Navigation Areas".

The direct calculations described in this note do not aim at replacing the classification notations, but aim at providing methodology for additional calculations which may be carried out for additional or complementary analysis. These complementary analyses may be used to provide additional information for the assessment of the hull reinforcements, the power propulsion, the mooring of offshore units… In particular, these analyses may be used to assess the adequacy of an ice class notation with a particular application not explicitly taken into account in the Rules.

This Guidance Note provides methodologies for evaluation of the loads due to the interaction between the ice and the structure. These loads may be used for the structural assessment of the ship or unit. This document is mostly based on API recommendation [5] and project of ISO standard [6].

1.3. Useful information to be collected

The data concerning ice properties and loads presented in this guidance note allow users to have a general overview of the mechanical properties of ice and the corresponding loads on a structure.

As in the different rules applicable, the environmental conditions (here mainly the ice properties) are to be submitted for the structural assessment. According to Bureau Veritas Offshore Rules [7], the following ice data concerning the site where the unit is going to operate are to be submitted:

- Ice conditions to be met (ice-floe, pack, ice bank, etc)
- Extreme level ice thickness, type (first or multi-year) and drifting speed
- Extreme ridge size and type (first or multi-year)
- Existence of ice islands and/or icebergs
- A table of the various ice thicknesses and types with associated probability of observation at reference site
- A ridge size classification with associated probability of observation at reference site

For level ice, the following properties are to be specified:

- Crushing strength
- Bending strength
- Buckling strength
- Brine volume.

Data concerning ice management are also to be submitted:

- Presence or not of icebreakers,
- Number and characteristics of icebreakers (size, ice class, etc),
- Operation limitations.

1.4. Units convention

Whenever possible, all values are given in the International System of Units (SI).

Forces	Newton	N
Lengths	Meter	m
Stresses	Pascal	Pa
Pressures	Pascal	Pa
Angles	Degrees	°
Temperatures	Degrees Celsius	°C
Energies	Joule	J
Time	Second	s

1.5. Symbols

The symbols used in this note are:

h : ice thickness, in m,

σ_f : flexural strength of the ice, in Pa, to be obtained from 3.2.5, or from model tests

α : angle between the sloping surface and the horizontal, in deg,

μ : friction coefficient between ice and structure, taken between 0.1 for freshly coated surfaces and 0.5 for rusty, rough surfaces.

$l = \left[\dfrac{E \cdot h^3}{12 \cdot (1 - \nu^2) \cdot \gamma_w} \right]^{1/4}$: characteristic length of floating ice sheet, in m,

E : effective modulus of ice, in Pa, taken in the range 1-3 GPa

ν : Poisson's ratio of ice, taken equal to about 0.33

γ_w : unit weight of water, usually taken equal to 1.025×10^4 N.m^{-3}.

2. CLASSIFICATION OF THE ICE

2.1. Macro description/glossary

2.1.1. Parameters of description

Sea ice can be described from two distinct parameters: its age and its form.

Sea ice can be broadly described as new ice, young ice, first-year ice, and old ice. These categories broadly reflect the age of the ice and include different forms and thicknesses of ice at various stages of development, as described in 2.1.2.

But sea ice can also be described according to the form of the ice, which can take numerous forms, as described in details in 2.1.3.

The following descriptions are extracts from a glossary of the National Oceanic and Atmospheric Administration (NOAA) [8], which was based on the standard definitions published by the World Meteorological Organization [9].

2.1.2. Macro description according to the age of the ice

The age of the ice is an important parameter as it influences the mechanical properties of the ice. In fact, the strength of the ice depends on the brine volume (volume of salt water in the ice) and this brine volume decreases significantly when the ice partially melts during the summer/autumn season, as the brine is drained through the porosity of the ice because of the gravity. Thus an old ice contains less brine and is notably stronger than a young ice.

The classification of the ice according to its age comes as follows:

- **New ice**

A general term for recently formed ice which includes frazil ice, grease ice, slush, shuga and nilas.

- Frazil ice

Frazil ice formation represents the first stage of sea ice growth. The frazil crystals are usually suspended in the top few centimeters of the surface layer of the ocean and give the water an oily appearance. In the open ocean the crystals may form, or be stirred to a depth of several meters by wave-induced turbulence.

- Grease ice

A later stage of freezing than frazil ice when the crystals have coagulated to form a soupy layer on the surface. Grease ice reflects little light, giving the surface a matt appearance, and behaves in a viscous fluid-like manner.

- Slush

Snow which is saturated and mixed with water on land or ice surfaces, or as a viscous floating mass in water after heavy snowfall.

- Shuga

An accumulation of spongy white lumps, a few centimeters across, which are formed from grease ice or slush and sometimes from anchor ice rising to the surface.

- Nilas

A thin elastic crust of ice, easily bending on waves and swell and under pressure, thrusting in a pattern of interlocking "fingers" (finger rafting). Has a matt surface and is up to 10 cm in thickness. May be subdivided into dark nilas and light nilas.

Dark nilas is < 5 cm thick and very dark in color. Light nilas is 5-10 cm thick and reflects proportionately more light than dark nilas, depending on its thickness.

- **Young ice**

Ice in the transition stage between nilas and first-year ice, 10-30 cm in thickness. May be subdivided into grey ice and grey-white ice.

- Grey ice

Young ice 10-15 cm thick. Less elastic than nilas and breaks on swell. Usually rafts under pressure.

- Grey-white ice

Young ice 15-30 cm thick. Under pressure more likely to ridge than to raft.

- **First-year ice**

Sea ice of not more than one winter's growth, developing from young ice; thickness (typically) 30 cm - 2 m. May be subdivided into thin first-year ice/white ice, medium first-year ice and thick first-year ice.

Thin first-year ice is 30-70 cm thick, medium first-year ice is 70-120 cm thick, and thick first-year ice is over 120 cm thick. First-year ice may be thicker than 200 cm when it is in the form of ridges.

- **Second-year ice**

Old ice which has survived only one summer's melt. Because it is thicker and less dense than first-year ice, it stands higher out of the water. In contrast to multi-year ice, summer melting produces a regular pattern of puddles. Bare patches and puddles are usually greenish-blue.

The regular pattern of puddles produced during the melt season is only a feature of Arctic se ice. Melt water does not usually accumulate on the surface of Antarctic sea ice.

Second-year ice is the most common form of old ice present in Antarctica.

- **Multi-year ice**

Old ice up to 3 m or more thick which has survived at least two summers' melts. Hummocks (hillocks of broken ice that have been forced up by pressure) even smoother than in second-year ice, and the ice is almost salt-free. Color, where bare, is usually blue. Melt pattern consists of large interconnecting irregular puddles and a well-developed drainage system.

2.1.3. Macro description according to the form of the ice

The form of the ice is also of great importance for the evaluation of the loads due to the ice environment that the ship or structure is going to evolve in.

- **Brash ice**

Accumulations of floating ice made up of fragments not more than 2 m across; the wreckage of other forms of ice. Brash is common between colliding floes or in regions where pressure ridges have collapsed.

- **Fast ice**

Sea ice which forms and remains fast along the coast, where it is attached to the shore, to an ice wall, to an ice front, between shoals or grounded icebergs. Fast ice may be formed in situ from sea water or by freezing of pack ice of any age to the shore, and it may extend a few meters or several hundred kilometers from the coast. Fast ice may be more than one year old and may then be prefixed with the appropriate age category (old, second-year, or multi-year).

- **Floe**

A floe is any contiguous piece of sea ice. Floes may be described in terms of several size categories:

- Giant: over 10 km across
- Vast: 2-10 km across
- Big: 500-2000 m across
- Medium: 100-500 m across
- Small: 20-100 m across
- Floes less than 20 m across are called cake ice

- **Iceberg**

A massive piece of ice of greatly varying shape, more than 5 m above sea-level, which has broken away from a glacier (or an ice shelf), and which may be afloat or aground. Icebergs may be described as tabular, dome-shaped, sloping, pinnacled, weathered or glacier bergs (an irregularly shaped iceberg).

Icebergs are not sea ice. They originate from the ice mass of the continent that has accumulated over many thousands of years. When they melt they add fresh water to the ocean

- **Pack ice**

Term used in a wide sense to include any area of sea ice, other than fast ice, no matter what form it takes or how it is disposed.

The pack can be described as very open (with an ice concentration of 1/10 to 3/10), open (4/10 to 6/10, with many leads and polynyas and the floes generally not in contact with one another), close (7/10 to 8/10, composed of floes mostly in contact), very close (9/10 to less than 10/10), and compact (10/10, with no water visible, called consolidated pack ice if the floes are frozen together).

- **Pancake ice**

Predominantly circular pieces of ice from 30 cm - 3 m in diameter, and up to 10 cm in thickness (unrafted), with raised rims due to the pieces striking against one another. It may be formed on a slight swell from grease ice, shuga or slush or as the result of the breaking of ice rind, nilas or, under severe conditions of swell or waves, of grey ice.

- **Rafting**

Pressure process whereby one piece of ice overrides another. Most common in new and young ice (cf. finger rafting).

Finger rafting is a type of rafting whereby interlocking thrusts are formed, each floe thrusting "fingers" alternately over and under each other. Common in nilas and grey ice.

Rafting plays an important role in increasing ice thickness within the Antarctic and Arctic pack. It is the dominant dynamic mechanism whereby floes reach between about 0.4 and 0.6 m thick in the early stages of ice development. Beyond this thickness, converging floes are more likely to form ridges than to raft.

- **Ridging**

The pressure process by which sea ice is forced into ridges.

A ridge is a line or wall of broken ice forced up by pressure. May be fresh or weathered. The submerged volume of broken ice under a ridge, forced downwards by pressure, is termed an ice keel.

2.2. Crystallographic description

2.2.1. Parameters of description

The properties of the ice depend on the meteorological and hydrodynamic conditions existing at the time of the formation. The crystallographic description deals with the crystal size and shape, and the crystallographic orientation. All these parameters and mainly the crystallographic orientation have a significant influence on the mechanical properties of the ice.

Note: The descriptions and figures used for the crystallographic description of the ice are mainly extracts from Bernard Michel's *Ice Mechanics* [10].

The crystal size is the grain size described according to the following:

- Fine: diameter less than 1 mm
- Medium: diameter 1 – 5 mm
- Large: diameter 5 – 20 mm
- Extra large: diameter greater than 20 mm
- Giant: grains with dimensions in meters.

The grain shapes are only described in qualitative terms and the crystallographic orientation described by the direction of the c-axis, which is the axis normal to the basal plane (see figure 1).

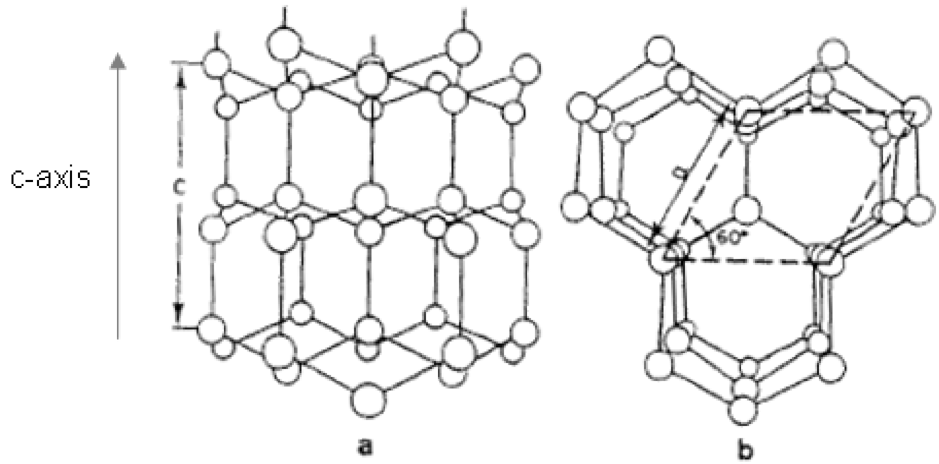


Figure 1: view perpendicular to the c-axis (a) and along the c-axis (b)

2.2.2. Primary ice

On a calm surface, the primary ice is an ice skim which grows horizontally in the supercooled layer and is a few tenths of a millimeter thick. In turbulent water it may consist of frozen frazil slush (fine spicules or plates of ice, suspended in water) which can be very thick.

- Primary ice of type 1 (P1)

This type of ice forms when the surface is calm and when temperature gradient is small. It is characterized by large, irregular grains and a crystallographic orientation of the c-axis preferred vertical. This type of ice is found in reservoirs, lakes and very calm areas of rivers but has not been observed in the sea. Under ideal conditions it is possible to observe giant sized grains.

- Primary ice of type 2 (P2)

This type of ice forms when surface is calm but temperature gradient is large. The grain size is varied, from medium to extra large. The grain shape varies from tabular to needle like and can be several cm long. The crystallographic orientation is random or vertically preferred superimposed on a random orientation. This type of ice is found in lakes, reservoirs, very calm areas of rivers and in sea ice.

- Primary ice of type 3 (P3)

This type of ice forms on an agitated surface and is nucleated from frazil. The grains are fine to medium and their shape is tabular. The crystallographic orientation is random and this type of ice can be found on lakes, reservoirs, rivers and is very general in the sea.

- Primary ice of type 4 (P4)

This type of ice is nucleated by snow and is very similar to the P3 ice.

2.2.3. Secondary ice

Secondary ice forms parallel to the heat flow which in most cases is perpendicular to the primary ice. Its structure is different from that of primary ice and may be in the form of columnar ice, the texture of which is entirely controlled by the primary ice. It can also be in the form of frazil slush or snow slush deposited under a primary ice layer, which after some time may become an integral part of the ice cover.

- Secondary ice of type 1 (S1)

This type of ice forms as a result of the conditions given for primary ice type P1 or P2. The grain size increases with depth and is usually large to extra large. Their shape is columnar and the crystallographic orientation is preferred vertical. It is found in lakes, reservoirs and rivers with low flow velocities but not in the sea.

- Secondary ice of type 2 (S2)

This type of ice forms in the same conditions that P2 ice but in this case the crystallographic orientation is preferred horizontal. Grain size increases more rapidly with depth than for S1 and starts at fine to medium to attain large and possibly extra large at the bottom of the columnar layer, particularly in Arctic Sea ice. It forms in rivers, lakes, reservoirs and in sea when conditions are steady enough.

- Secondary ice of type 3 (S3)

This type of ice forms at the bottom of thick ice sheets and covers. It develops from a preferred horizontal orientation to one with several of the c-axes nearly parallel.

- Secondary ice of type 4 (S4)

This type of ice is made of congealed frazil. The grains are equiaxed and tabular in shape. Their size is fine to medium and depends on the age of the originating frazil. The crystal boundaries are irregular and the crystallographic orientation is random. The frazil is formed in turbulent water and is swept under the secondary ice cover. It is found in rivers as well as reservoirs and lakes fed by turbulent waters and at sea when frazil slush is produced.

- Secondary ice of type 5 (S5)

This type of ice is made from drained congealed frazil slush. The grains range from fine to medium size and are angular in shape. This ice is found where water has drained through the ice cover leaving the slush to be refrozen.

2.2.4. Superimposed ice

Superimposed ice always forms on top of the primary ice and is caused by flooding of the ice cover from any water source. If there is snow on the ice surface, snow ice will form.

- Superimposed ice of type T1 – snow ice

The grain size range from fine to medium, the shape is round to angular depending on the age of the snow, and the grains are equiaxed with a random orientation. This ice is formed when water saturates a snow deposit which then freezes.

- Superimposed ice of type T2 – drained snow ice

The grains range from fine to medium size and are well rounded. The layer is homogeneous ad randomly oriented. This ice is found where water levels can vary rapidly, draining a previously saturated snow cover which then refreezes.

- Superimposed ice of type T3 – superimposed layered ice

This type of ice is made of layers of columnar ice which have formed on top of the original primary ice.

2.2.5. Agglomerate ice

This ice is an agglomeration of various ice types and forms which have refrozen. The grain size can vary from fine to extra large and the shape from equiaxed to tabular to columnar with the crystal boundaries regular to angular in shape. The orientation can vary from random to a preferred orientation. It is frequently found in areas of turbulent flow or rapids in rivers and ice accumulation in the sea. It can also be found in rafted or ridged ice.

2.2.6. Most common crystallographic types of ice encountered

It is to be noted that for ice in sea environment, the S2 is the most probable type of ice. However, in Saint Laurent bay, other type may be encountered.

3. MECHANICAL PROPERTIES OF ICE

3.1. Mechanical properties of ice monocrystals

3.1.1. Elastic moduli

For a non isotropic crystal, Michel [10] gives the following relationship for the elastic behavior:

$$\varepsilon_i = S_{ij}\sigma_j$$

$$\sigma_i = C_{ij}\varepsilon_j$$

Where S_{ij}, in Pa^{-1}, and C_{ij}, in Pa, are matrix coefficients representing compliance and stiffness constants respectively, and i and j take integral values from 1 to 6 inclusive. For hexagonal crystals, and for ice, there are only six non-zero independent elastic constants, namely S_{11}, S_{12}, S_{13}, S_{33}, S_{44} and S_{66} with the corresponding C_{ij}.

These notations can be taken as follows in relation to the orientation of the main optical axis:

- S_{11} gives the extension perpendicular to the c-axis due to a longitudinal tensile stress also perpendicular to the c-axis, in the same direction.
- S_{12} gives the extension perpendicular to the c-axis due to a longitudinal stress also perpendicular to the c-axis, but also perpendicular to the direction the extension is measured.
- S_{13} gives the extension perpendicular to the c-axis due to a longitudinal stress along to the c-axis and is equal to the extension along the c-axis due to a tensile stress perpendicular to it.
- S_{33} gives the extension along the c-axis due to a tensile stress along it.
- S_{44} gives the shear strain on a plane containing the c-axis due to a shear stress in the same plane.
- S_{66} gives the shear strain on a plane perpendicular to the c-axis due to a shear stress in the same plane. It is derived from the other constants.

Various formulae can be found in the literature for the calculation of the S_{ij}, Michel [10] gives a formulation in function of the temperature θ of the ice.

The values of the dynamic moduli vary with the direction of loading. For any direction the compliance constant S_α, in Pa^{-1}, can be obtained from the following equation:

$$S_\alpha = S_{33} \cdot \cos^4 \alpha + S_{11} \cdot \sin^4 \alpha + (S_{44} + S_{13}) \cdot \sin^2 \alpha \cdot \cos^2 \alpha$$

where the S_{ij} are the compliance constants, in Pa^{-1}, quoted above and α is the angle, in degrees, between the direction of loading and the crystal c-axis.

The stiffness constant C_α of ice, in Pa, is the reciprocal to the corresponding compliance constant. It varies by about 30% depending upon α and its minimum value is obtained with an angle of 45°, as shown on figure 2.

It can be seen from figure 2 that the maximum value for the stiffness constant of an ice monocrystal would be 12 GPa, but this value might be 30% over the effective value on some directions.

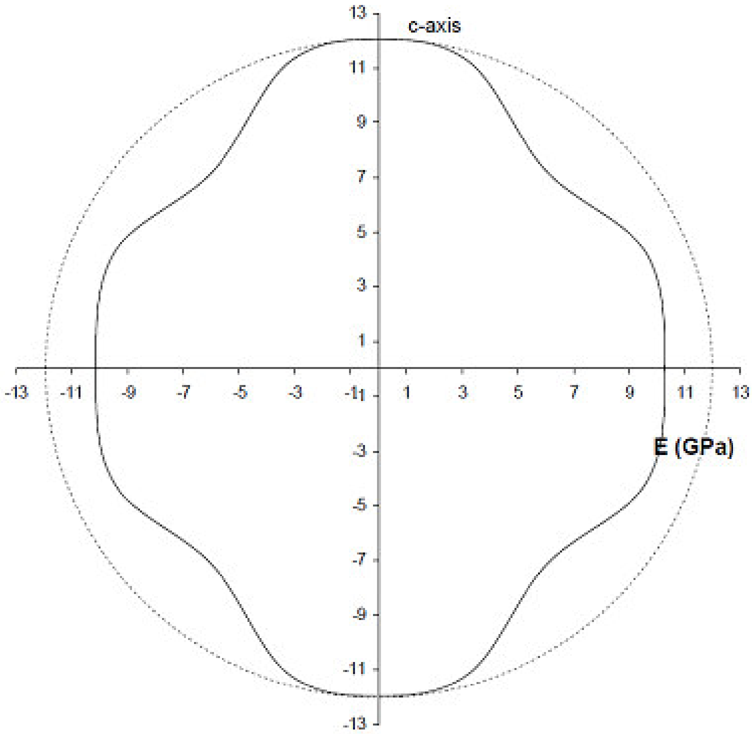


Figure 2: Stiffness constant of an ice monocrystals a function of the stress orientation

3.1.2. Tensile strength

According to Michel [10], the tensile strength of an ice monocrystal is linked to the surface energy of the ice, which is the energy necessary to initiate a fracture, in J.m^{-2}, and given by:

$$\sigma_{CV} = 0.109 J.m^{-2}$$

If the crystal is considered as a cylinder of height H, the condition of fracture is given by:

$$\tau \geq 2\sqrt{\frac{\sigma_{CV} \cdot G_{66}}{H}}$$

where

τ : minimum shear stress necessary to initiate a crack, in Pa,

σ_{CV} : surface energy defined above, in J.m^{-2} (or N.m^{-1}),

G_{66} : shear modulus, in Pa, and is obtained from the value of S_{66} :

$$G_{66} = 1/S_{66} = 3.374 \cdot \left(1 - 0.89 \times 10^{-3}\theta\right) \times 10^{9} Pa$$

For tests in uniaxial tension, the values of τ and σ, both in Pa, in a basal plane making an angle α with the normal to the loading direction are given, from simple stress analysis, by:

$$\tau = \sigma_{\alpha} \cdot (\sin 2\alpha)/2$$

$$\sigma = \sigma_{\alpha} \cdot (1 + \cos 2\alpha)/2$$

Where σ_{α} is the uniaxial strength of an ice sample where the basal plane has an orientation α .

And the condition of fracture now becomes for σ_{α}, in Pa:

$$\sigma_{\alpha} \geq 4 \cdot \frac{1}{\sin(2\alpha)} \cdot \sqrt{\frac{\sigma_{CV} \cdot G_{66}}{H}}$$

The lowest value of σ_α is for $\alpha = 45°$, which gives the value of σ_{45}, in Pa:

$$\sigma_{45} = 4 \cdot \sqrt{\frac{\sigma_{CV} \cdot G_{66}}{H}}$$

And the condition of fracture can be written:

$$\sigma_\alpha \geq \sigma_{45} \cdot \frac{1}{\sin(2\alpha)}$$

The tensile strength of S1 ice at -10°C has been measured by Carter and Michel [10], in function of the angle α and the results are given in figure 3. For $\alpha = 45°$ the average value of σ_{45} is $\sigma_{45} = 22 \times 10^5 \, Pa$. This allows to trace the previous relationship in full line and to asses its validity for $\alpha \in [15°;75°]$.

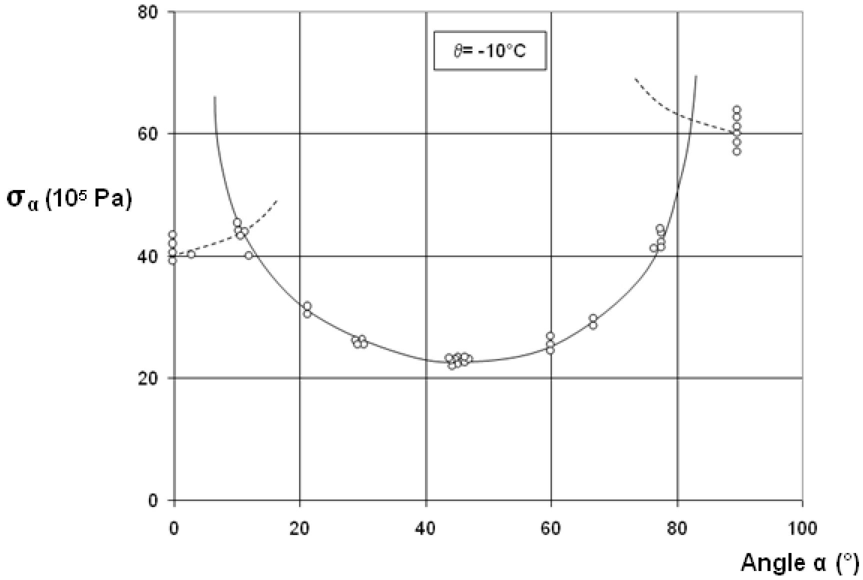


Figure 3: Variation of tensile strength of S1 ice in function of angle α of basal plane with the normal to the loading direction, according to Michel [10]

3.1.3. Compression strength

Under compressive stresses, the major difference compared to tensile strength is that the formation of a crack needs an additional energy because of the cohesive strength of the ice C_i, in Pa.

Thus the total shear stress that produces cracking is given, in Pa, by:

$$\tau' = \tau^* + C_i$$

where τ^* is the minimum shear stress necessary to initiate a crack, in Pa, as defined in [3.1.2] by:

$$\tau^* \geq 2\sqrt{\frac{\sigma_{CV} \cdot G_{66}}{H}}$$

And the condition of fracture then becomes:

$$\sigma_\alpha' \geq 2\tau' \cdot \frac{1}{\sin(2\alpha)}$$

The compressive strength of S1 ice at -10°C has been measured by Carter and Michel [10], in function of the angle α and the results are given in figure 4.

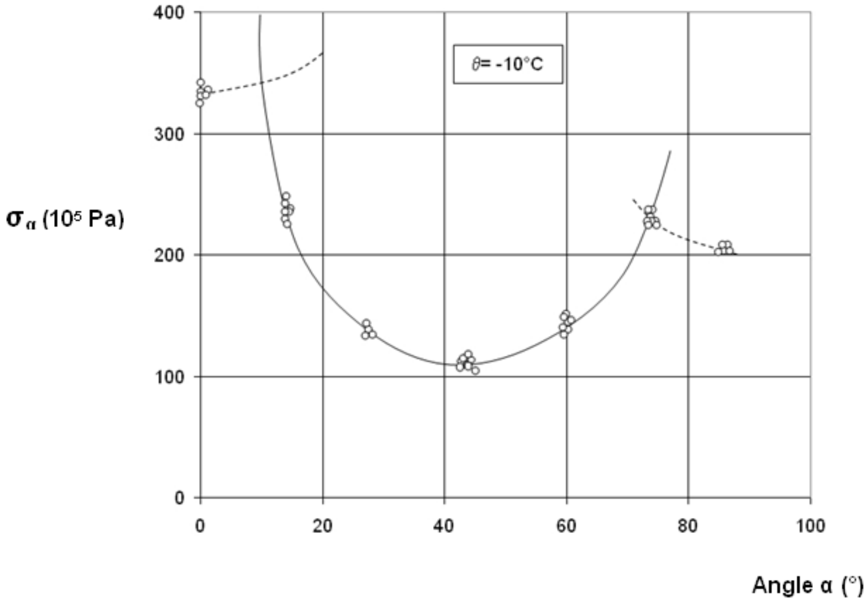


Figure 4: Variation of compressive strength of S1 ice in function of angle α of basal plane with the normal to the loading direction, according to Michel **[10]**

3.2. Mechanical properties of polycrystalline ice

3.2.1. Introduction

Since real ice is always composed of various ice types of an important range of sizes and orientations, its mechanical properties are significantly different from these of monocrystals. Ice is a unique material. In the temperature range under which it is normally encountered, it is very close to its melting point. Ice can creep with very little applied stress, or it can fracture catastrophically under a high strain rate.

Both the porosity within the ice and the grain structure influence the mechanical properties of the ice. The porosity attributable to brine (salt water inside the ice) and air pockets affects the ice properties.

The brine volume ratio v_b, in parts per thousand (‰), is obtained from the following relation, as discussed in the US Army publication *Ice Engineering* **[11]**:

$$v_b = S_i \left(0.532 + \frac{49.185}{|T|} \right)$$

Where S_i : salinity, in ‰,

T : temperature of the ice, in °C,

The porosity ascribable to air can be obtained from the following relation, according to *Ice Engineering* [11]:

$$\frac{V_a}{V} = 1 - \frac{\rho}{\rho_i} + \rho \cdot S_i \cdot \frac{F_2(T)}{F_1(T)}$$

where: V_a : volume of air, in m^3,

V : bulk volume, in m^3,

ρ : measured bulk density of ice containing salt and air, in Mg.m^{-3},

ρ_i : density of pure ice, in Mg.m^{-3},

S_i : salinity of ice, in ‰,

$F_1(T)$, $F_2(T)$: functions of temperature derived from a phase equilibrium table and given in figure 5.

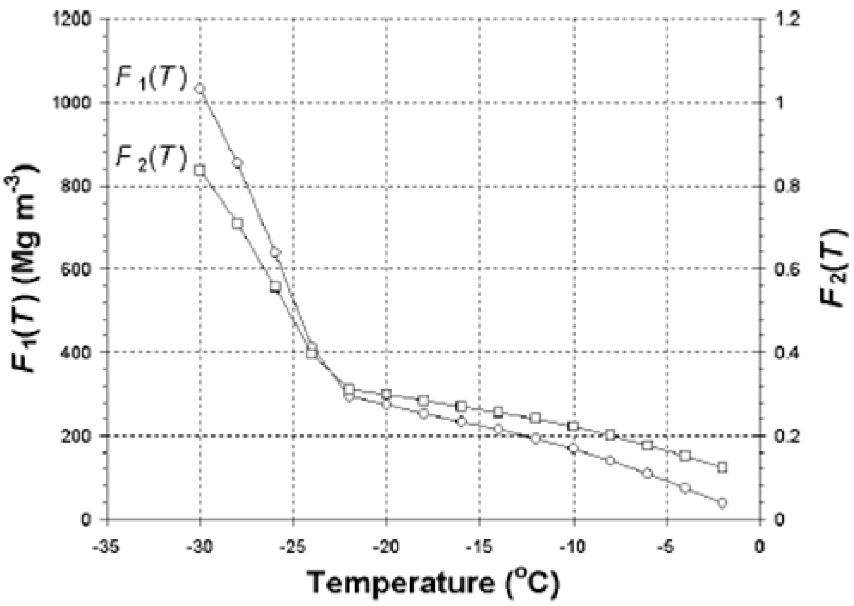


Figure 5: $F_1(T)$ and $F_2(T)$ with respect to temperature, from U.S Army [11]

3.2.2. Elastic modulus

Ice deformation involves elastic and creep processes, and the large-scale modulus is usually discussed in terms of an “effective modulus” that incorporates these processes.
This modulus is function of loading rate, temperature, grain size and grain type. The values of elastic modulus range from approximately 2 GPa at low frequency loading to a high frequency value of 9 GPa, according to *Ice Engineering* [11].
Elastic modulus of ice depends also on the total void volume v_T, according to ISO draft document [6]:

$$E = E_0 \cdot \left(1 - \sqrt{v_T}\right)^4$$

where v_T : total ice volume fraction, equal to the volume of void over the volume of ice

E_0 : reference elastic modulus, in Pa,

E : effective elastic modulus, in Pa.

3.2.3. Compression strength

Values of the uniaxial compressive strength for ice range from 0.5 to 20 MPa, according to *Ice Engineering* [11]. The strength is a function of strain rate, temperature, grain size, grain structure, and porosity.
Analyses of strength measurements have shown that the strength increases with strain rate, up to a rate of 10^{-3} s^{-1}, whereupon the strength generally decreases at higher strain rates because of brittle fracture.

In the lower strain rate range below 10^{-3} s^{-1}, the compressive strength of freshwater ice is given by:

$$\sigma_c = 212 \cdot \dot{\varepsilon}^{0.34} \cdot \left(3.07 \times 10^4 \cdot \dot{\varepsilon}^{0.34}\right)$$

where σ_c : compressive strength of freshwater ice, in MPa,

$\dot{\varepsilon}$: strain rate, in s^{-1}.

For sea ice, the following equations for compressive strength were derived from an analysis of over 400 sample tests. These equations are:

$$\sigma_c = 37 \cdot \dot{\varepsilon}^{0.22} \cdot \left[1 - \left(\frac{v_T}{270}\right)^{0.5}\right]$$ for horizontally loaded columnar sea ice

$$\sigma_c = 160 \cdot \dot{\varepsilon}^{0.22} \cdot \left[1 - \left(\frac{v_T}{200}\right)^{0.5}\right]$$ for vertically loaded columnar sea ice

$$\sigma_c = 49 \cdot \dot{\varepsilon}^{0.22} \cdot \left[1 - \left(\frac{v_T}{280}\right)^{0.5}\right]$$ for granular sea ice.

where σ_c : compressive strength of sea ice, in MPa,

$\dot{\varepsilon}$: strain rate, in s^{-1},

v_T : total porosity in the ice (brine and air), in parts per thousand.

The range of strain rate for these equations is 10^{-7} to 10^{-4} s^{-1}. Above this strain rate, the ice can experience brittle failure with compressive strengths exhibiting a wide range of variability.

If the maximum compressive strength is to be obtained, $\dot{\varepsilon}^{0.22}$ can be replaced by the factor 0.22. Also, the equation for granular ice can be used for sea ice since it is its most common type.

As guidance, values basically range from 3 to 6 MPa in the Baltic Sea and 1.5 to 5 MPa in most parts of the Arctic region (values up to 8 MPa can be observed in the high Arctic, where multi-year ice is predominant). The dependence with the strain rate is higher than with the temperature.

3.2.4. Tensile strength

The tensile strength is nearly independent of the strain rate in the range of: $\dot{\varepsilon} = 10^{-5}$ s^{-1} to 10^{-3} s^{-1}.

The tensile strength in the growth (vertical) direction of columnar grained ice is about twice as high as in the horizontal direction. The tensile strength depends also on the brine volume of the sea ice, as can be found in the *ISO standard 19906* [6].

Based on the data shown in Table 1, the tensile strength can be expressed, in MPa, as:

$$\sigma_t = 2.2 \cdot \left(1 - \sqrt{\frac{v_b}{0.31}}\right)$$ in the horizontal direction

$$\sigma_t = 1.0 \cdot \left(1 - \sqrt{\frac{v_b}{0.14}}\right)$$ in the vertical direction

where v_b is the brine volume fraction.

Temperature	Ice type	Ice Salinity	Action direction	
			Vertical	Horizontal
-10° C	Sea Ice	4 – 6 ‰	1.7 MPa	0.9 MPa
-10° C	Baltic Ice	0.2 ‰	2.2 MPa	1.0 MPa

Table 1: Ice tensile strength, according to ISO 19906 draft [6]

3.2.5. Flexural strength

The flexural strength is generally lower than the compressive strength.
Measurements on freshwater ice range from 0.5 to 3 MPa, with an average of 1.73 MPa, for temperatures less than –5°C according to *Ice Engineering* [11]. There is very little temperature or strain rate dependence, but there is a wide scatter in the measured flexural strength with higher values from smaller samples.

For sea ice, the following dependence of the flexural strength on the brine volume was obtained from the compilation of results of over 900 flexural strength measurements:

$$\sigma_f = 1.76 \cdot e^{-5.88\sqrt{v_b}}$$

where σ_f : flexural strength of sea ice, in MPa,

v_b : brine volume fraction, in ‰.

For sea ice, values can range from 0.5 to 0.7 MPa in the Baltic Sea in winter and they can range from 0.3 to 0.6 MPa in most parts of the Arctic (but values are up to 0.9 MPa in the high Arctic, where multi-year ice is predominant). If site specific tests or small-scale tests are conducted, the size effect on flexural strength should be accounted for.

3.2.6. Toughness

The fracture toughness K_{IC} of ice depends on the size of the piece of ice that is being fractured, the length of the crack, the temperature, the composition of the ice, and the action rate. A commonly quoted value of K_{IC} for ice at laboratory scale is 100 kPa.m$^{0.5}$. In-situ fracture tests on sea ice samples larger than 3 m have shown that K_{IC} is about 250 kPa.m$^{0.5}$.

4. ICE-STRUCTURE INTERACTIONS

4.1. Introduction

Once the characteristics of the ice are known it is possible to determine what is the load exerted by the ice on a given structure. The load on the structure is determined by the failure of the ice, which can be one of the following modes: failure by crushing, failure by flexural bending, failure by buckling or failure by splitting. The present chapter describes the different modes of failure of the ice against structures and the resulting load can be determined by considering the lowest mode of failure

4.2. Contact pressure and effective pressure

It is important to distinguish between the contact pressure acting over the ice–structure contact area and the effective pressure. Because the actual contact area between a structure and an ice sheet is less or equal than the nominal contact area (product of ice thickness h and structure width w), the contact pressure is higher or equal than the effective pressure. The effective pressure corresponds roughly to an average pressure. This is described in Figure 6.

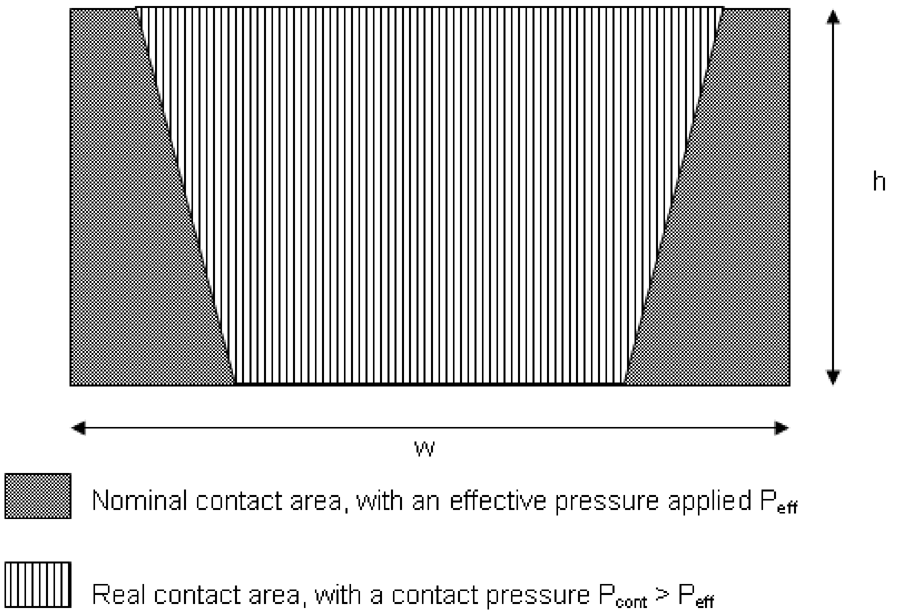


Figure 6: Difference between contact and effective pressures

4.3. Crushing failure

Ice in nature exists close to its melting temperature, and its temperature affects its properties. Ice at lower temperatures is stronger and also has more brittle characteristics. The indentation speed also influences the ice properties. Ice creeps at low rates of loading, and it fails in a brittle manner at high loading rates.

At low indentation rates, the ice deforms in creep, resulting in full contact and uniform pressure at the interface. At high rates of ice indentation, ice crushes continuously in a brittle manner, resulting in non-simultaneous, partial contact and non-uniform pressure over the nominal contact area. At intermediate rates, the interaction between structural deformation and an advancing ice sheet produces alternating ductile and brittle crushing, resulting in ice force records taking a saw-tooth form.

4.3.1. Ductile deformation of the ice

In *Ice Engineering* [11], results of small-scale indentation tests on freshwater columnar ice indicate that the effective pressure for ductile (creep) deformation of ice at strain rates between 10^{-8} s^{-1} and 5×10^{-4} s^{-1} is given by:

$$p_c = C \cdot m \cdot k \cdot \sigma_c \cdot \left(\dot{\varepsilon} / \dot{\varepsilon}_0 \right)^{0.32}$$

where p_c : effective average pressure, in MPa,

C : indentation factor, equal to 2.97,

m : shape factor for the structure on which the ice is acting, taken equal to 1.00 for a plane contact area, 0.90 for a circular contact area, 0.81 for a 120° wedge, 0.73 for a 90° wedge, 0.65 for a 60° wedge.

k : contact factor, taking account of type of contact between ice and structure. It reaches 1 for the first peak force and then decreases to 0.6 for the steady-state pressure after the peak force.

σ_c : crushing strength of the ice, in MPa, obtained from tests when possible, or from 3.2.3,

$\dot{\varepsilon}$: empirically defined strain rate, in s^{-1},

$$\dot{\varepsilon} = \frac{v}{4 \cdot D}$$

v : indentation rate, in m.s^{-1},

D : indenter width, in m,

$\dot{\varepsilon}_0$: reference strain rate, equal to 5×10^{-4} s^{-1}

There is no confirmation whether the indentation factor and the empirical definition of strain rate ($v/4D$) will remain applicable for very large aspect ratios (D/h). Such confirmation is perhaps impossible because of creep buckling of floating ice sheets against wide structures at a lower effective pressure than that required for in-plane creep indentation.

4.3.2. Brittle crushing

For edge indentation into floating ice sheets, the main characteristics of brittle crushing are the line-like contact in the middle third of the ice sheet thickness, the non-simultaneous contact in different parts of the contact line, and the non-uniform pressure in the contact area. The results of full-scale measurements of ice forces, and medium- and small-scale tests indicate that the effective pressure for brittle crushing and for high aspect ratio (D/h) is in the range of 1 and 3 MPa, which is less by a factor of three to four in comparison to the maximum pressure that develops at the high end of speed range for ductile deformation of ice over small areas or small aspect ratios (D/h). The reason for the reduction in effective pressure can be attributed to the actual contact area during brittle crushing being much smaller than full contact during ductile deformation of ice.

The following is an expression to estimate ice force F on a structure of width D for continuous brittle crushing of ice of thickness h at high indentation rates:

$$F = A_r \cdot p_{b,c} \cdot D \cdot h$$

where $p_{b,c}$: effective pressure (1.5 to 2 MPa) for brittle crushing of ice, in MPa,

A_r : empirical factor to account for the aspect ratio effect of high effective pressure over small aspect ratios.

$$A_r = \left(\frac{5h}{D} + 1 \right)^{0.5}$$

D : width of the structure, in m,

h : height of the ice cover, in m,

F : force on the structure, in MN.

4.3.3. Empirical approach

Figure 7 shows plots of effective pressures measured during small- and medium-scale tests, ship ramming, and large-scale field monitoring of ice forces versus nominal contact area. Others have also compiled the so-called pressure-area plots. In plotting these data, no regard is given to the speed of indentation into the ice. There is a large scatter in the data on effective pressures for contact areas less than 5 m^2, and this can be attributed to variations in indentation speed. The results of small-scale tests show that there is a decrease in effective pressure with increasing indentation speed, even when the contact area is kept constant. Lack of scatter in the data for effective pressure for areas greater than 100 m^2 can be attributed to brittle crushing having been active at high indentation speeds and the creep buckling of floating ice sheet against wide structures preventing the development of high indentation pressure at low ice speeds.

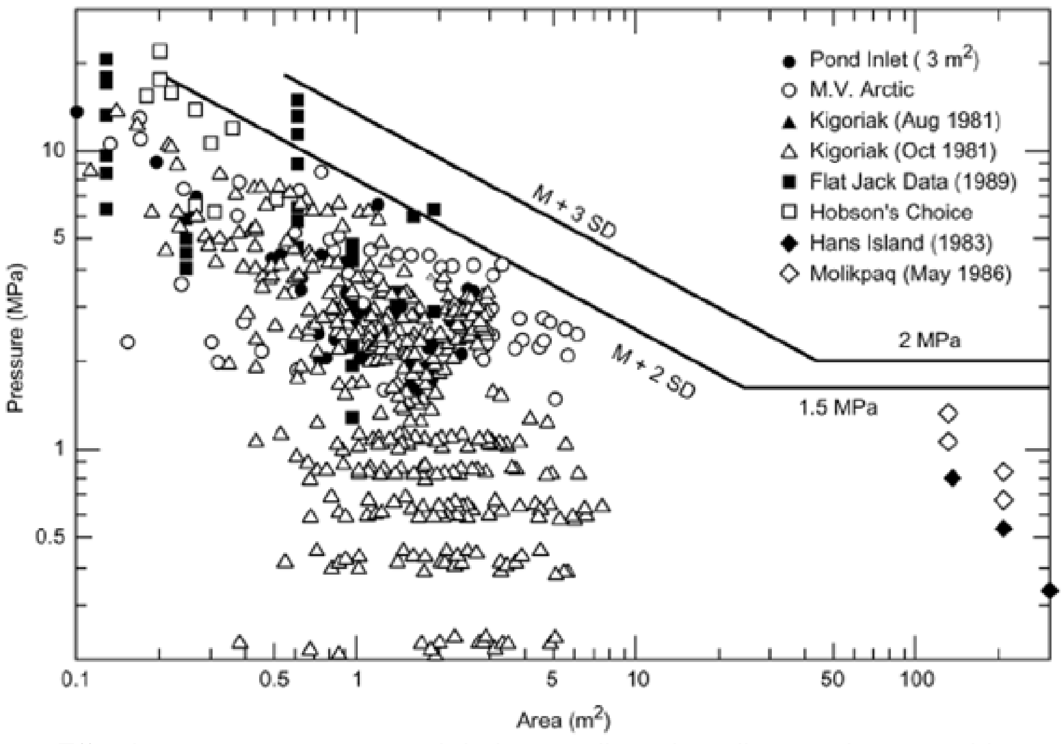


Figure 7: Effective pressures measured during small- and medium-scale tests, ship ramming, and large-scale field monitoring of ice forces versus nominal contact area

The effective pressure, measured during crushing of first-year ice against the 100-meter-wide Molikpaq structure at ice speeds greater than 100 mm.s^{-1}, is in the range of 1 to 2.5 MPa. Effective pressures in the range of 1–3 MPa have also been measured on indenters during small-scale tests in the same velocity range. These two observations indicate that, when continuous brittle crushing is active, the effective pressure is independent of the nominal contact area. Because high contact pressure can act over a small area resulting from ductile deformation of ice, the trend in the upper bound of effective pressure versus contact area (Figure 6) shows a decrease in effective pressure with increasing contact area. Though this trend is known as a scale effect in the literature, the real reason for the decrease in effective pressure with increasing contact area is the possibility of high pressure developing over a small area because of ductile deformation and crushing of the ice in the brittle mode over a large contact area or high aspect ratio (D/h).

Two lines in Figure 7 , labelled as M+2SD and M+3SD, signify trend lines of mean (M) plus two and three standard deviations (SD) of the data, respectively. Both of these models have been recommended in *API Recommended Practice 2N* **[5]**. The recommended pressures in the *Canadian codes for offshore structures* **[12]** are similar.

4.4. Floe splitting

During ice crushing, a crack often forms at the contact zone and may propagate away from the ice flow at very high speed. When the crack opens, a splitting failure occurs and it causes a lower load than crushing load.

In *API Recommended Practice 2N* **[5]**, the floe splitting pressure p_{fs} , in kPa, which acts over a length b centrally located along the width W of the floe, both in m, is given by:

$$p_{fs} = K_{1C} \cdot \frac{\sqrt{L}}{b} \cdot F\left(\frac{W}{L}\right)$$

where K_{1C} : fracture toughness, in kPa.m^{0,5}

L : length of the floe, in m,

$F\left(\frac{W}{L}\right)$ is a function of $\frac{W}{L}$.

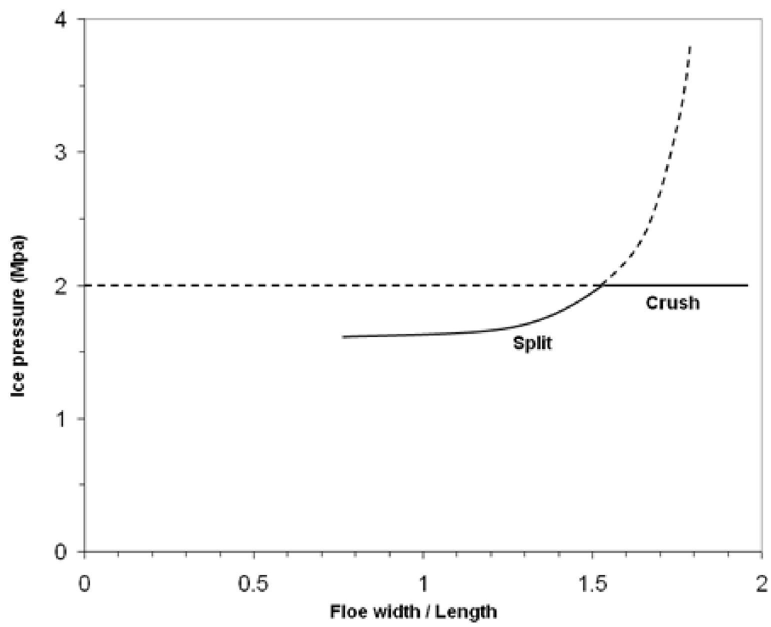


Figure 8: Floe splitting

Consider an example of $K_{1C} = 100$ kPa.m^{0,5}, $L = 2000$ m and $b = 10$ m. The pressure p_{fs} (in MPa) to split the floe is shown in figure 8 as well as crushing pressure of 2 MPa. This example predicts that the peak forces due to floes less than 3000 m wide will be limited by splitting.

4.5. Bending failure

When the ice sheet encounter a conic or a sloped structure, the maximum bending force acting on the structure may be evaluated considering the bending strength of the ice sheet.

4.5.1. 2D Bending failure

To study the bending of the ice in 2D, a semi-infinite beam is used, with elastic springs to take into account the hydrostatic stiffness (figure 9).

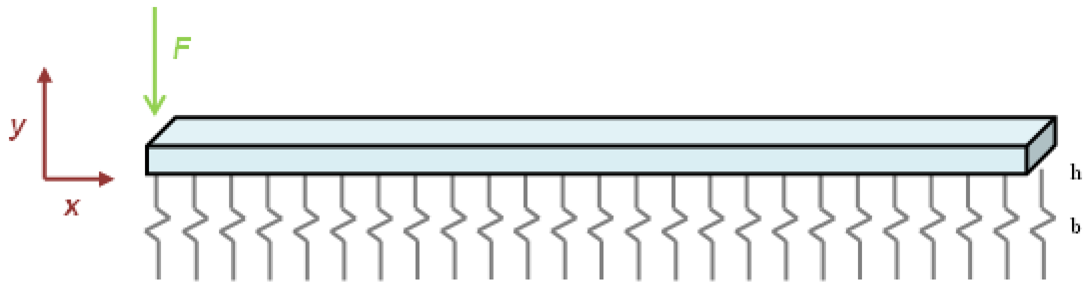


Figure 9: Beam on elastic springs

The equations of equilibrium of the internal forces lead to the following equation:

$$EI_z \frac{d^4 w}{dx^4} + kw = 0$$

$$k = \rho_w g b$$

To solve this equation it can be assumed:

$$\beta = \left(\frac{k}{4EI_z} \right)^{1/4} = \frac{1}{\sqrt{2} L_b}$$

This parameter is consistent with the length L_b which is the characteristic length of the beam:

$$L_b = \left(\frac{EI_z}{k} \right)^{1/4}$$

Assuming $l = \sqrt{2} L_b$, the solution is:

$$w(x) = e^{x/l} \left(C_1 \sin \frac{x}{l} + C_2 \cos \frac{x}{l} \right) + e^{-x/l} \left(C_3 \sin \frac{x}{l} + C_4 \cos \frac{x}{l} \right)$$

C_1, C_2, C_3 et *C_4* are integration constants. Then the bending moment and the shear force are:

$$M_z = -EI_z \frac{d^2 w}{dx^2}$$

$$T_y = -EI_z \frac{d^3 w}{dx^3}$$

Boundary conditions lead to the determination of the constants C_1, C_2, C_3 et C_4 :

$$\begin{cases} w(x \to \infty)) = 0 \\ T_y(x=0) = -F \\ M_z(x=0) = 0 \end{cases} \Rightarrow \quad \begin{array}{l} C_1 = C_2 = C_3 = 0 \\ C_4 = -\dfrac{2F}{kl} \end{array}$$

$$w(x) = -\frac{2F}{kl}\cos\frac{x}{l}e^{-x/l}$$

$$M_z(x) = -Fl\sin\frac{x}{l}e^{-x/l}$$

$$T_y(x) = -F\left(\sin\frac{x}{l} - \cos\frac{x}{l}\right)e^{-x/l}$$

The bending moment and shear force distributions are shown on the figure 10.

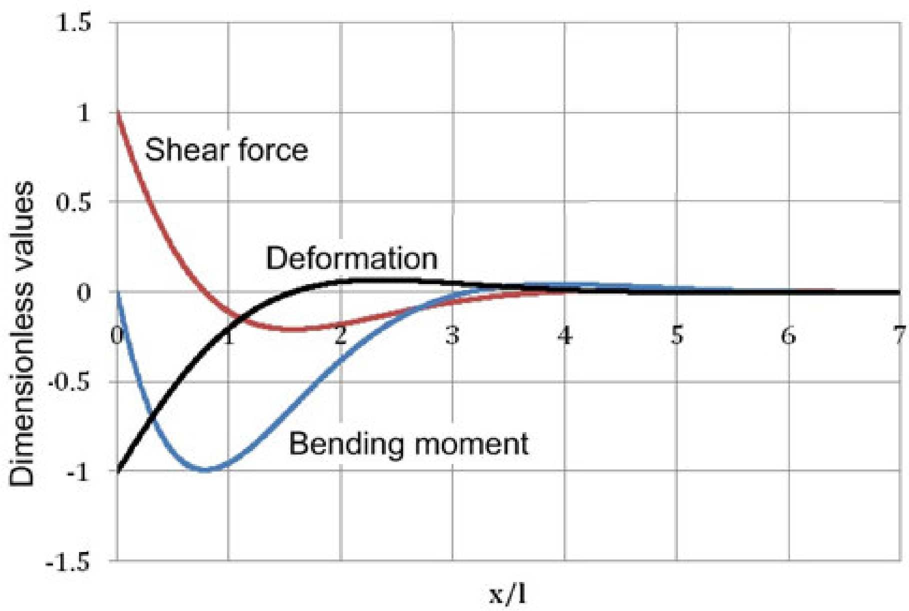


Figure 10: Deformation, shear force and bending moment on x-axis

It was verified that the shear was not critical for the rupture of the ice sheet, and thus the failure is made by bending, at the location where the moment is maximum.
The moment of inertia of a rectangular beam and the bending stress are:

$$I_z = \frac{bh^2}{12}$$

$$\sigma_x = -\frac{6M}{bh^2}$$

The maximum bending moment is located at the abscissa x such that:

$$x/l = \pi/4$$

$$x_{max} = \frac{\pi}{4}\sqrt{2}L_b$$

$$M_{max} = -F\sqrt{2}L_b\sin\frac{\pi}{4}e^{-\pi/4}$$

$$\sigma_{max} = \frac{6\sqrt{2}FL_b}{bh^2}\sin\frac{\pi}{4}e^{-\pi/4}$$

The force per unitary length may be calculated as follows:

$$\frac{F}{b} = \frac{\sigma_f h^2}{6\sqrt{2}L_b \sin\frac{\pi}{4}} e^{\pi/4}$$

Figure 11 shows the necessary force for the failure of the ice sheet by bending for different ice strength (0.5-1 MPa) and Young modulus (2.5-10 GPa).

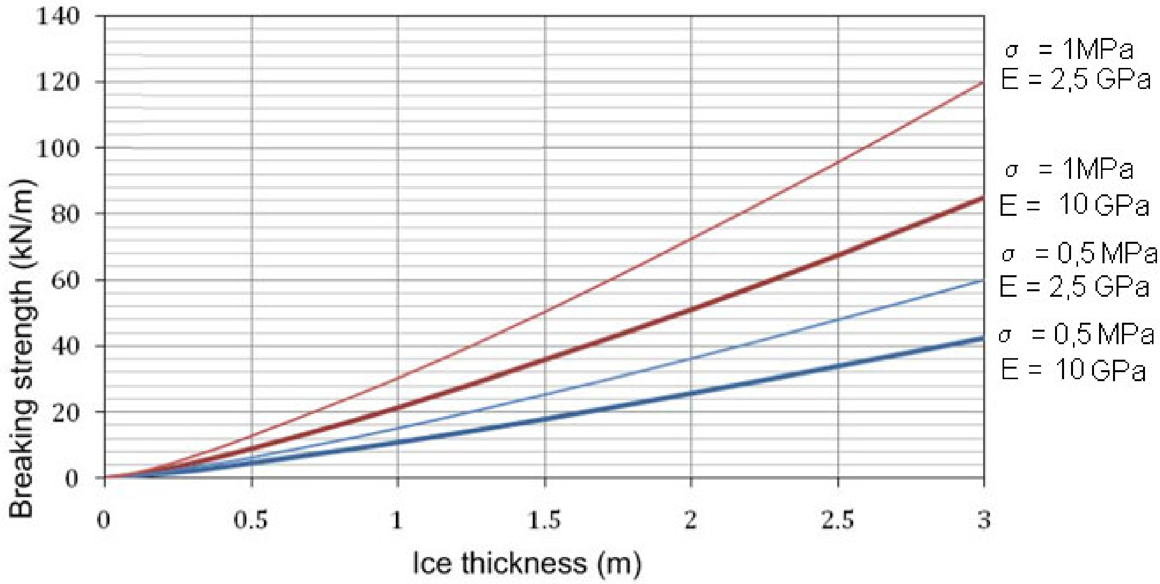


Figure 11: Distribution of force per unit of breadth for failure of ice by bending for different values of bending strength σ (0.5-1 MPa) and Young modulus E (2.5-10 GPa)

For information, Table 2 provides some values of x (abscissa of maximum bending moment) for different ice thicknesses:

h(m)	0.5	1.0	2.0	3.0
x (m)	10.9	18.3	30.9	41.7

Table 2: Position of the failure versus the ice thickness

4.5.2. Interaction with a sloped structure

Figure 12 shows forces during an interaction of a floating ice sheet of thickness h being pushed against a wide sloping surface at an angle α with the horizontal, as can be found in the *API Recommended Practice 2N* [5] or in *Ice Engineering* [11].

Under the assumption that there is no moment acting on the floating ice sheet, the minimum vertical force C_V per unit width, in $N.m^{-1}$, required to break a floating ice sheet that is pushed against the structure, is given by the following equation:

$$C_V \geq \frac{\sigma_f \cdot h^2}{6 \cdot l \cdot e^{-\pi/4} - h \cdot \tan(\alpha + \arctan\mu)}$$

In the case of an ice sheet pushed against a structure, at the time the ice fails in bending, the horizontal reaction force per unit width C_H , in N.m^{-1}, from the structure on the ice sheet is given by the following equation:

$$C_H = C_V \cdot \tan(\alpha + \arctan\mu) = \frac{\sigma_f \cdot h^2 \cdot \tan(\alpha + \arctan\mu)}{6 \cdot l \cdot e^{-\pi/4} - h \cdot \tan(\alpha + \arctan\mu)}$$

These two formulae give information about the maximum force that can occur on the structure, at the time the ice fails by bending, but nothing concerning the local pressure. Even if the global mode of failure is bending, locally the ice fails by crushing before a larger ice sample fails by bending.

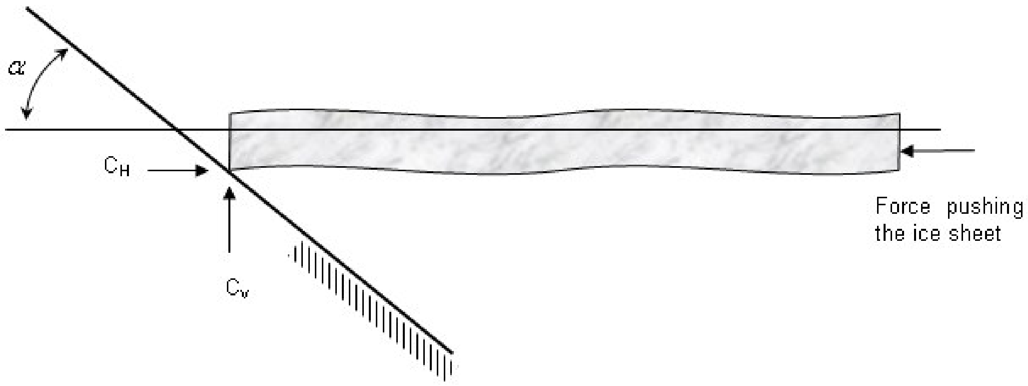


Figure 12: forces during an interaction of a floating ice sheet of thickness h being pushed against a wide sloping surface at an angle α with the horizontal.

4.5.3. Bending failure in 3D

The 3-dimensional study is based on the same principles, but the problem solved by using finite elements analysis and not analytically (figure 13).

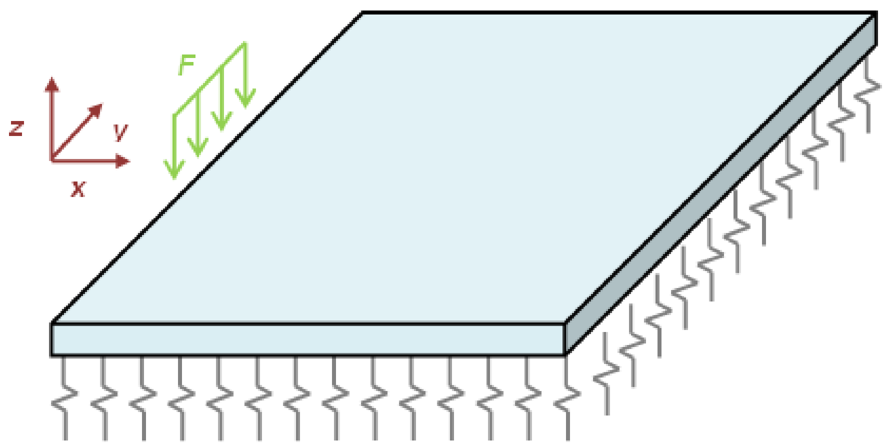


Figure 13: Plate on elastic springs

The characteristic length is now expressed by:

$$L_s = \left(\frac{Eh^3}{12\rho_w g(1-\nu^2)} \right)^{1/4}$$

And it can be seen that the place where the ice fails does not depend on the applied force and is expressed by $x = \frac{\pi}{4} L_s$, as shown on figure 14.

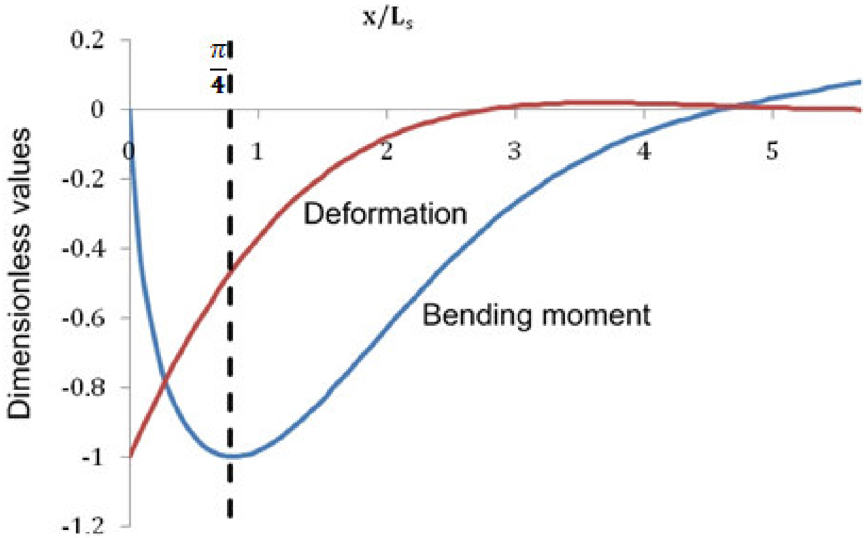


Figure 14: Deformation and bending moment on x-axis

The criterion in shear was never overcome.

From figure 15, showing the bending moment of axis x and the deformation, both along the y-axis, it is seen that the moment is maximum where the force is applied, which can lead to a failure of the ice in this part.

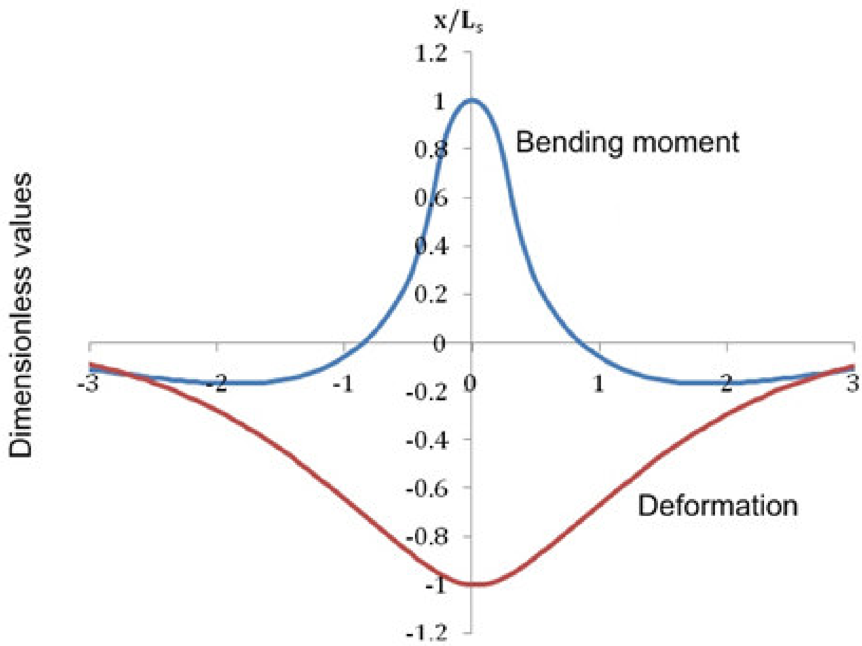


Figure 15: Bending moment of axis x and deformation, on y-axis

The chronology of the ice failure can be summed up in the figure 16.

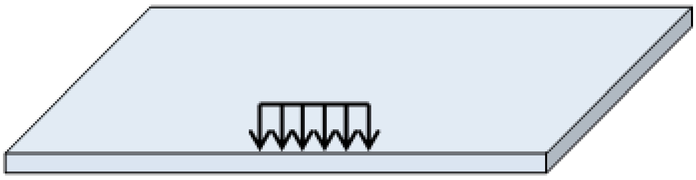
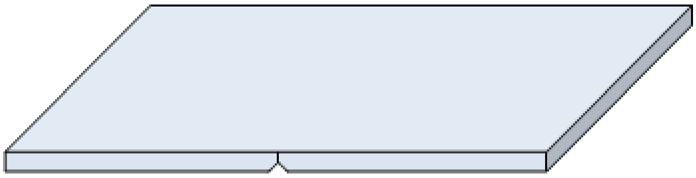
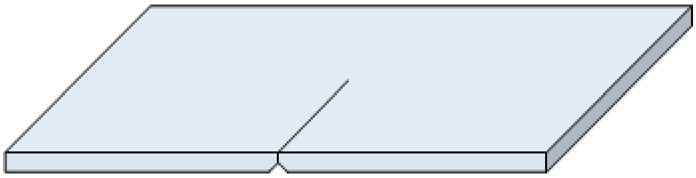
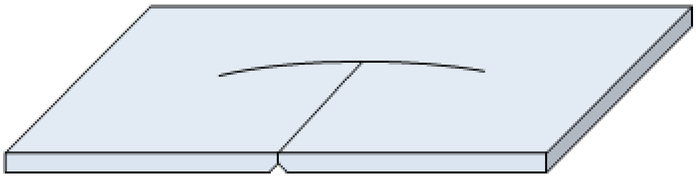
	Vertical force applied on the edge of the ice plate.
	First failure of the ice on the lower boundary of the plate due to the bending moment of axis x.
	Propagation of the failure of the ice, in a radial direction.
	Circumferential failure at a certain distance of the edge. Ultimate failure of the ice.

Figure 16: Chronology of the failure of the ice

This phenomenon can be seen on the figure 17, where the two types of cracks (radial and circumferential) are shown:

Figure 17: Failure of the ice

4.5.4. Comparison of the 2D and 3D effects

If the vertical force is no more concentrated on a single point but distributed over a width b as shown on Figure 18, the profile of the bending moment of axis x changes significantly in function on this width b, as shown in figure 19.

When the width of the applied force grows, the ice doesn't fail in a radial direction first but directly in a circumferential direction, and at a distance x from the edge of the plate.

In such a case the length of the circumferential failure can be expressed by $l_c = b + \pi x = b + \frac{\pi^2}{4} L_s$.

This behavior is thus rather close of what goes on with the 2D beam and it is of interest to compare 2D and 3D results (see Fig 20).

The link between 2D and 3D results for the force to be applied to break the ice can be expressed by the formula:

$$F_{3D} = F_{2D}\left(1 + \frac{\pi^2}{4}\frac{L_s}{b}\right) = \frac{\sigma_f h^2}{6\sqrt{2} L_s \sin\frac{\pi}{4}} e^{\pi/4}\left(b + \frac{\pi^2}{4} L_s\right)$$

This formula is the same that what is found in Croasdale [13].

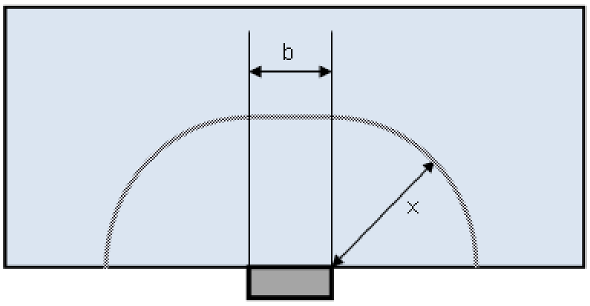


Figure 18: Plate with a vertical distributed force over a width b

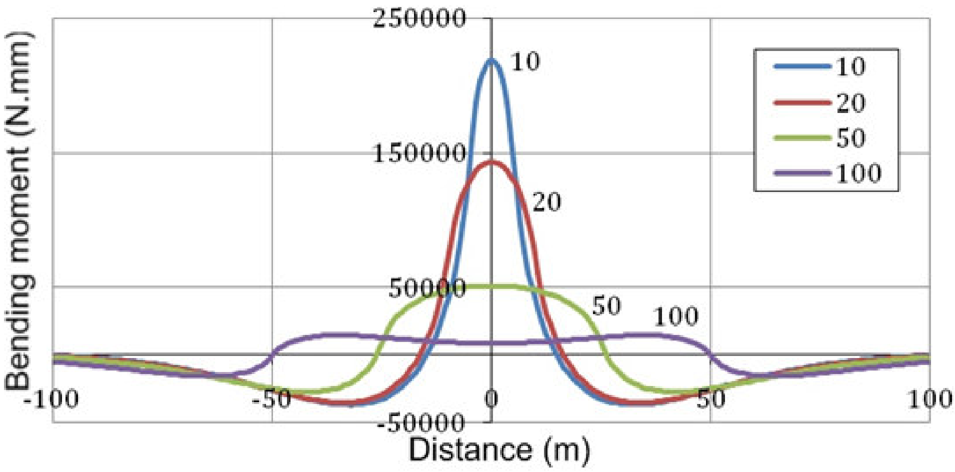


Figure 19: Bending moment of axis x, along y-axis

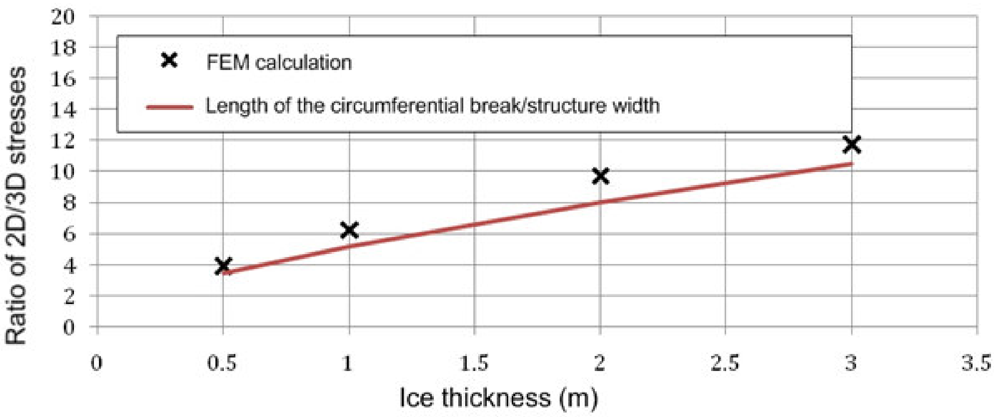


Figure 20: Comparison between the ratio of 2D and 3D stresses and the ratio of the length of circumferential failure over structure width

4.6. Buckling failure (2D, 3D and comparison)

4.6.1. Buckling failure in 2D

To study the buckling of the ice in 2D, a semi-infinite beam is used, with elastic springs to take into account the hydrostatic stiffness (figure 21).

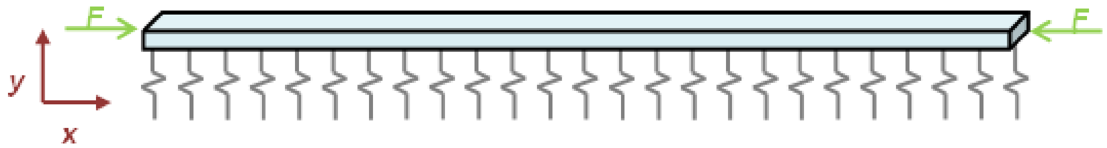


Figure 21: Beam on elastic springs

The equation of the deformation is the following:

$$EI_z \frac{d^4 w}{dx^4} + F \frac{d^2 w}{dx^2} + kw = 0$$

Considering that $w(x) = e^{\lambda x}$, it comes:

$$EI_z \lambda^4 + F\lambda^2 + k = 0$$

Let's state that $\alpha = \left(\frac{k}{EI_z}\right)^{1/4} = \frac{1}{L_b}$ and $\gamma = \frac{F}{2\sqrt{kEI_z}}$.

The solutions of the differential equation depend on the sign of the discriminant of the differential equation, and thus on the values of γ.

The fixity conditions of the model allow setting up a 4 x 4 linear system of equations that gives the integration constants of the differential equation.

Then the critical dimensionless load $\frac{F}{bkL_b^2}$ can be represented on figure 25, where:

$$L_b = \left(\frac{EI_z}{k}\right)^{1/4}$$

It can be seen that the evolution of the critical load depends on the conditions of fixity of the beam and that the limit when the length increases is equal to 1 when the extremity of the beam is free, and 2 in any other case. Moreover these graphics are in accordance with the ones of the API [5].

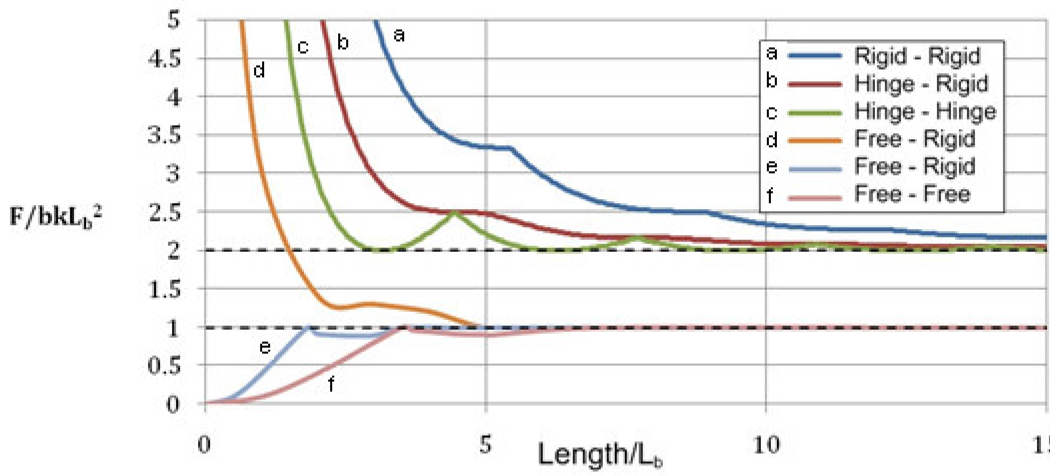


Figure 22: Critical buckling load versus length of the beam for various fixity conditions

4.6.2. Elastic wedge buckling

The buckling of a floating semi-infinite wedge (See figure 23) using beam theory has been determined analytically and the buckling of a semi-infinite wedge using plate theory has been determined using finite elements. Both results agree for angles of 45° or less.
The dimensionless buckling load for a floating wedge-shaped semi infinite ice sheet is shown on figure 11 as a function of R/l where R is the truncated distance of the wedge measured from the apex (see figure 23) and l is the characteristic length. The results are given for three different boundary conditions and for the wedge angles of 2°, 90 ° and 180°.
Theses results can be approximated by:

$$\frac{P}{B \cdot \gamma_w \cdot l^2} = C + \frac{D}{R/l}$$

where C and D are coefficients which are given in table 3 for various wedge angles and boundary conditions of the wedge.

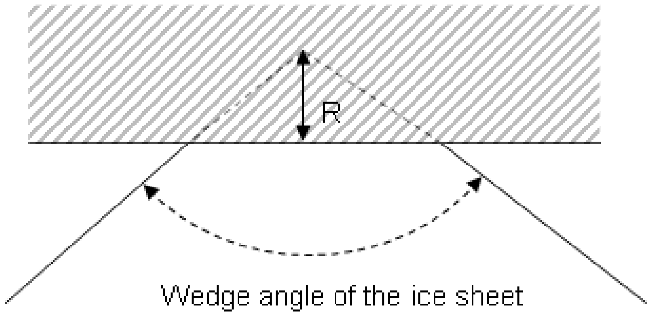


Figure 23: Definition of the wedge angle and the truncated distance

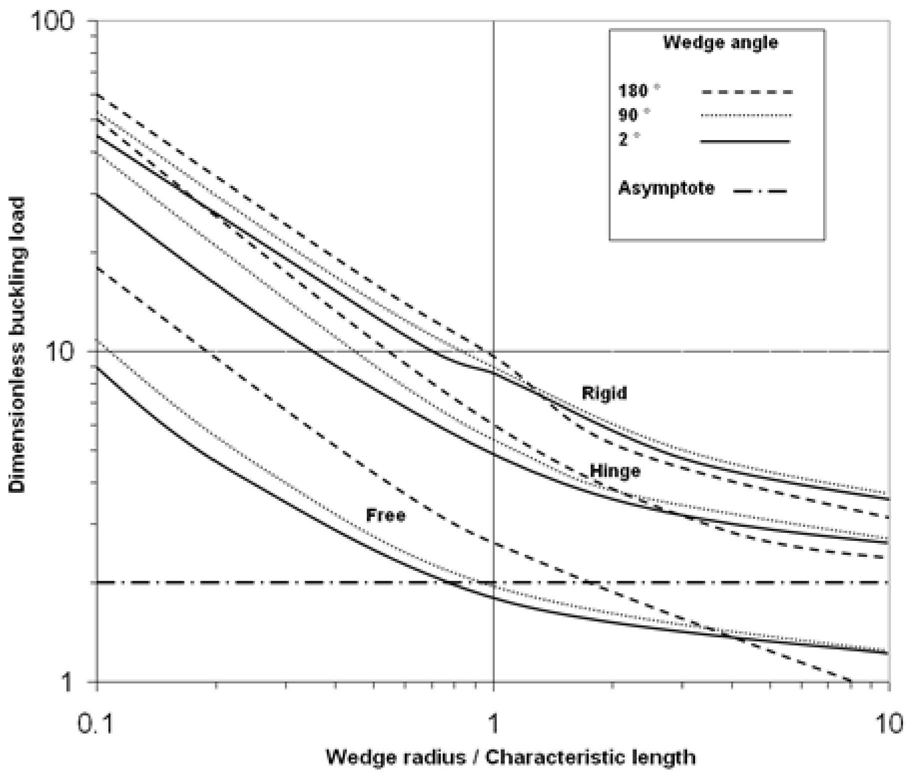


Figure 24: Wedge buckling dimensionless factor, from API [5]

Angle	Boundary Condition					
	Free		Hinged		Rigid	
	C	D	C	D	C	D
2 °	0.96	0.80	2.11	2.76	2.57	4.47
30 °	1.00	0.82	2.20	3.11	2.55	4.70
90 °	0.95	1.01	2.04	3.78	2.35	5.34
150 °	0.84	1.36	1.81	4.30	2.08	5.83
180 °	0.81	1.66	0.75	4.67	2.04	6.05

Table 3: Buckling coefficients

4.6.3. Buckling failure in 3D

The 3D study is based on the following model (figure 25):

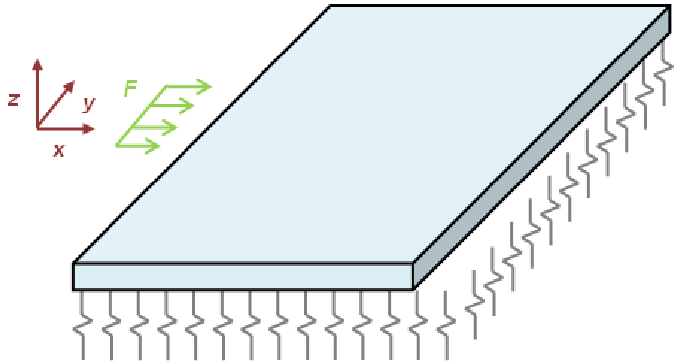


Figure 25: Plate on elastic springs

The complexity of the problem leads to use a finite elements method to solve it (figure 26):

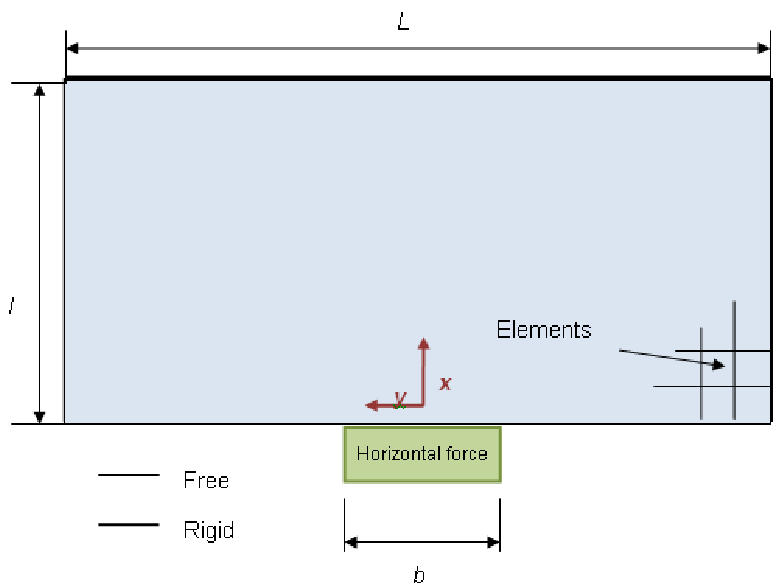


Figure 26: FEM model used for the buckling of the plate

The critical load F is made dimensionless by dividing it by kL_s^3 and it is expressed by unit of width by dividing it by bkL_s^2, where the characteristic length is still given by:

$$L_s = \left(\frac{Eh^3}{12\rho_w g(1-\nu^2)} \right)^{1/4}$$

As shown in figure 27, the results obtained from the finite element methods fit with what can be found in the literature, in Michel [10]:

$$\frac{F}{kL_s^3} = \frac{b}{L_s} + \frac{3.32}{1+\frac{b}{4L_s}}$$

Or in Sodhi [14]:

$$\frac{F}{bkL_s^2} = 0.81 + \frac{1.66}{\frac{b}{2L_s}}$$

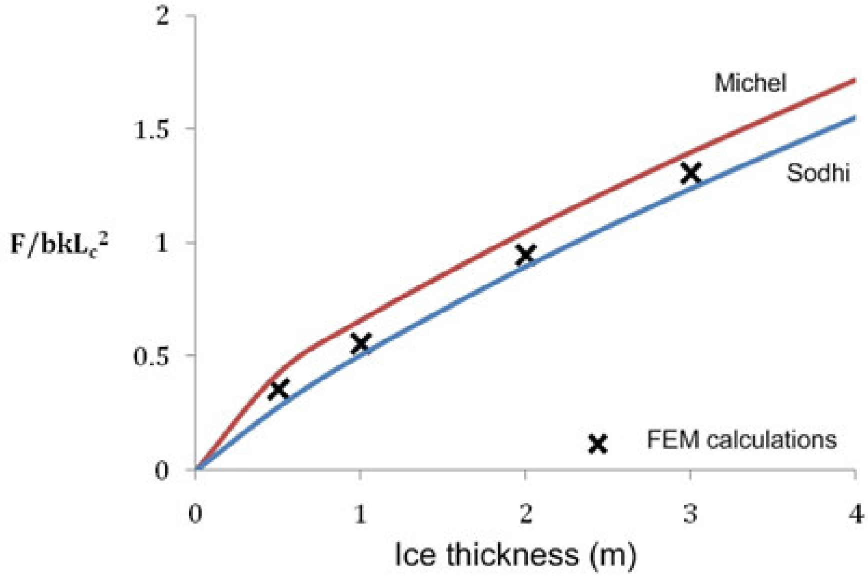


Figure 27: FEM results compared with different formulae

4.6.4. Comparison of the 2D and 3D effects

Figure 28 shows another comparison between 2D and 3D results.

The comparison shows that the dimensionless load per unit of width issued either by finite elements calculations or Michel formulae is close to 2.5 when the width of the applied force is close to 0.

And when the width of the applied force rises, the critical load tends to be the same than for the 2D formulation, which seems logical.

Figure 29 shows the evolution of the critical load F in function of the ratio of the structure width b on the ice thickness h. The curves are plotted for various ice thicknesses (0.5 m; 1 m and 2 m) and for two values of Young's Modulus (2.5 GPa and 10 GPa).

This figure shows that the critical load decreases when b/h >> 1, which means that it is for the widest structures that buckling failure is likely to occur.

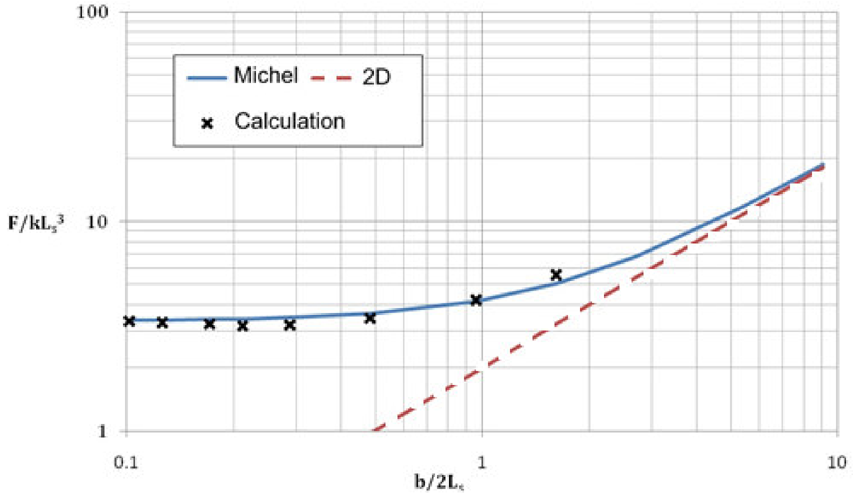


Figure 28: Comparison of 2D and 3D results with Michel formulae

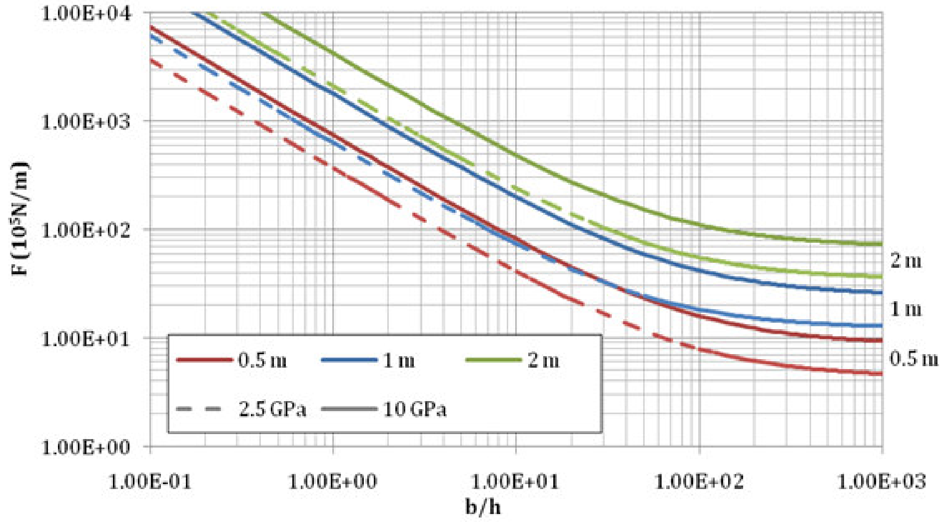


Figure 29: Critical load in function of the ratio b/h

4.6.5. Influence of Ridges

A Ridge, is a rectilinear conglomeration of ice fragments formed by pressure at the contact line between ice floes (Figure 30), usually along earlier existing cracks and leads or at the boundary between ice floes of different age. Ice ridges can also form as a result of direct fracturing of ice fields of thick ice at very strong pressures. The underwater portion of a ridge is termed an ice keel and above water portion is named the sail.

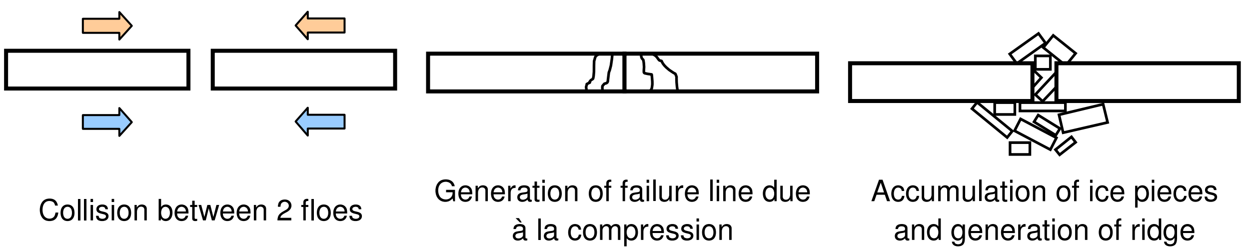


Figure 30 : Formation of a ridge by compression

For hydrostatic balance, the keel is always larger than the sail.

The influence of the ridges may be taken into account also on the bending and buckling strength of the level ice with finite element model.

The presence of ridge may significantly increase the strength of the level ice.

Figure 31 shows examples of model with ridges. This exemple shows a ridge perpendicular to the ship sailing direction. However, the ridges are to be taken into account in different directions, i.e. parallel perpendicular but also with an angle to the ship sailing direction. The case with a ridge perpendicular is not the most conservative case.

The presence of the ridge led to the modification of bending stress in the ice sheet as shown on Figure 32.

The ice sheet will fail in way of the maximum bending stress. I. e. the ice sheet may break before of after the ridge. The ridge itself will not be broken by bending.

The influence of different positions of ridge was tested. The presence of the ridge leads to an increase of the vertical force on the structure of the vessel.

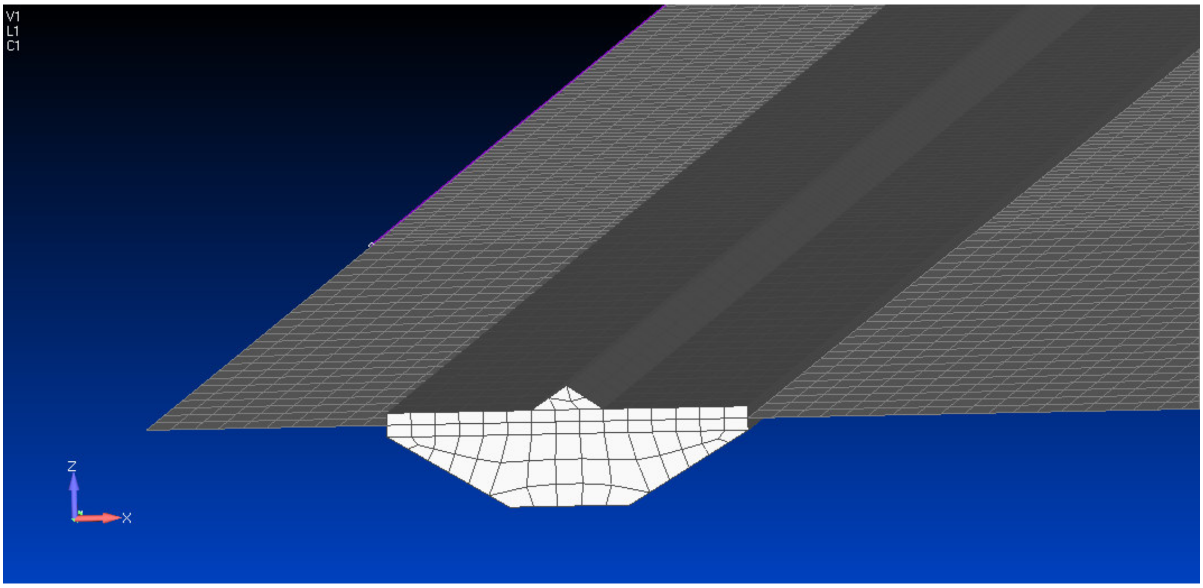


Figure 31: Model of the ice sheet with a ridge

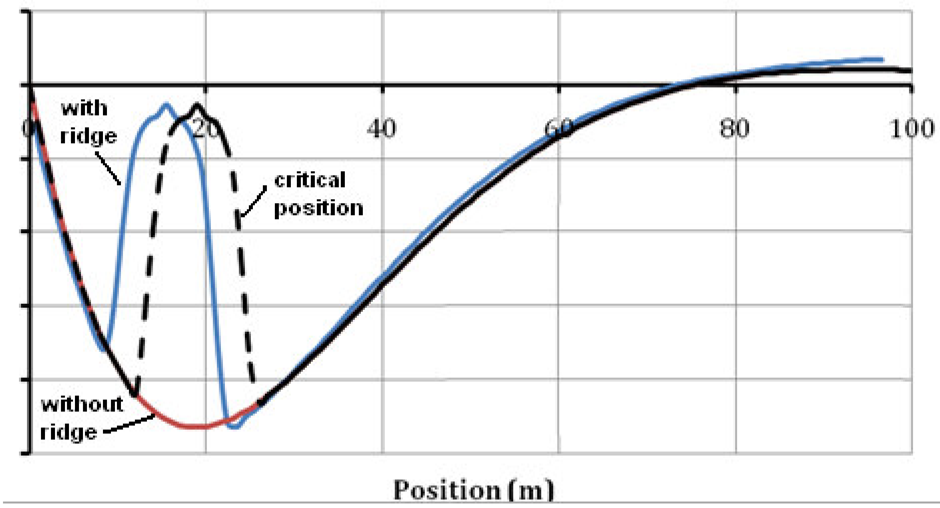


Figure 32: Bending Stress distribution along the ice sheet with a ridge

4.7. Limit between the different failure modes

4.7.1. Limit between bending and buckling modes

Concerning the limit between bending and buckling failure modes of the ice, the two main parameters of influence are the angle of inclination of the structure as well as the friction coefficient of the ice on the structure.

In figure 33, the horizontal line corresponds to the critical buckling load while the curved lines correspond to the critical bending load for various values of the friction coefficient (0; 0.1 and 0.2) and of the limit bending stress of the ice (0.5 MPa and 1 MPa).

Figure 33 shows that the buckling mode of failure is independent of the inclination angle while the bending failure mode is very dependant on this angle. But the most important parameter might be the friction coefficient between the ice and the structure and a reasonable value for this coefficient is around 0.1.

This means that for most steel structures, an angle of inclination greater or equal to roughly 10 deg is highly advisable to ensure that the ice will break through a bending mode of failure and thus with a lot less efforts on the structure.

4.7.2. Limit between bending and crushing modes

Concerning the limit between the bending and crushing failure modes, the parameter that governs the failure is the size of the contact area on the structure. The mechanism of failure is explained on figure 34. This critical area can be analytically expressed by comparing the critical bending force and the force applied on the structure by crushing

The figure 35 presents the two modes of failure in function of the contact area for an ice thickness of 3 m, a friction coefficient of 0.1 and an angle of inclination of 10 degrees.

Figure 35 shows that the critical area is equal to 0.05 m², which means that the failure of the ice by crushing is a marginal phenomenon and the ice rapidly fails by bending.

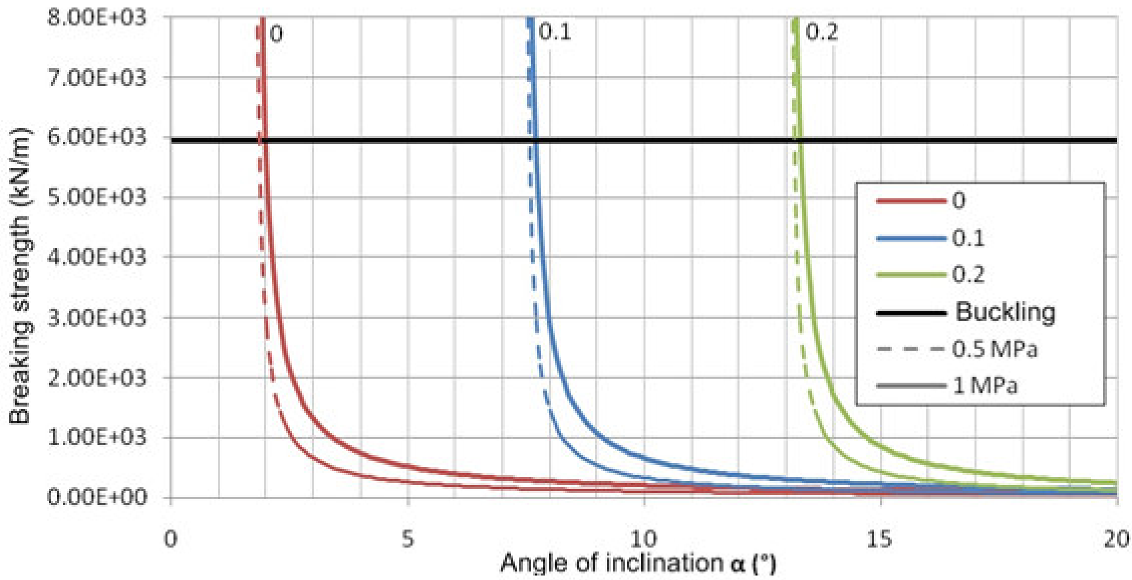


Figure 33: Critical load in function of the inclination angle for various friction coefficients

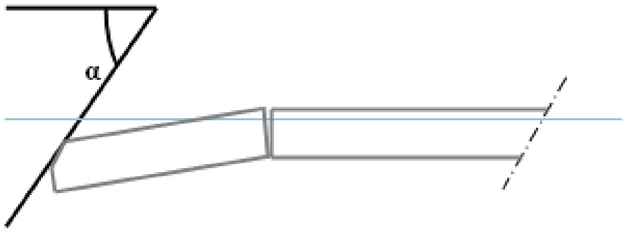
	At first, the ice encounters the structure.
	The ice starts by failing by crushing as long as the contact area is under a given value. The pressure on the contact zone is relatively high but very concentrated.
	Once the contact area has reached a critical value, which means that the total force acting on the ice sheet has reached the critical bending force, the ice breaks at a certain distance of the structure.

Figure 34: Ice failure mechanism on an inclined structure

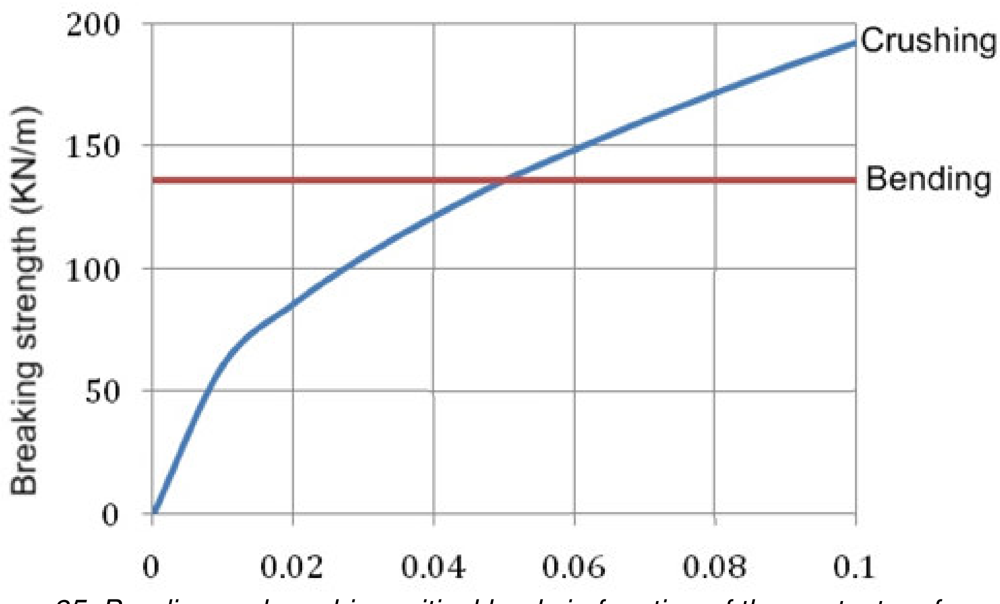


Figure 35: Bending and crushing critical loads in function of the contact surface area

4.8. Conclusion

These previous studies allowed performing a validation of the formulae found in the literature for the estimation of the forces generated by the ice on the structures. They also allowed knowing what the different modes of failure that can occur are and how they occur, in function of the parameters of the problem (thickness of the ice, inclination of the structure, friction coefficient …)

These formulae do not lead to a sharp knowledge of the loadings on the structure but in a first analysis they can provide beneficial data to the designer.

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Achevé d'imprimer sur les presses d'Ediprint
53940 Saint Berthevin (France)
Septembre 2010